Self-Employment and Development Policies*

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Abstract

We study how the efficacy of development policies– such as job guarantee programs, unemployment insurance, and micro-finance– depends on the prevalence of low-earning self-employed individuals. To this end, we develop a new general equilibrium occupational choice model that is consistent with the behavior and composition of self-employment. Our model differs from previous work by allowing unemployment risk to shape the selection of agents into self-employment. Models that rely only on financial frictions are at odds with crucial features of self-employment in developing economies. In particular, the concentration of self-employed agents among the lowest earners of the economy, and their willingness to take on salaried jobs when offered to them. These features support the prevalence of *subsistence entrepreneurs* in developing economies, who play a critical role in shaping policy responses. Their willingness to accept jobs at market wages leads to a muted response of wages to labor demand shocks as in job guarantee programs. Additionally, offering small unemployment benefits reduces subsistence entrepreneurship, increasing productivity and output. In contrast, micro-finance exacerbates this phenomenon, reducing productivity.

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Business ownership is regarded as a booster of innovation, economic dynamism, and growth. However, a cross-country comparison of self-employment rates offers a puzzling outlook. Self-employment rates are higher in developing countries than in developed ones.[1] This apparent contradiction is due to the large fraction of self-employed individuals in the developing world engaged in small and unproductive enterprises, which provide subsistence income (Schoar, 2010; Poschke, 2013a,b). The reality of self-employment in developing countries is one of street vendors, not one of growing start-ups.

Aiming to spur economic growth, policymakers advocate for development policies that alleviate barriers to the entry and growth of firms. These policies are intended to promote business ownership and improve allocative efficiency, thus reducing the productivity gaps behind differences in standards of living across countries (see Hsieh and Klenow, 2009, 2014; Banerjee and Moll, 2010; Restuccia and Rogerson, 2013; Midrigan and Xu, 2014). However, as we expand on below, the specifics of the mechanisms that lead individuals to opt into self-employment play a crucial role in shaping the aggregate response to policy.

In this paper we evaluate the effect of development policies in environments where *subsistence entrepreneurship* is prevalent. We study the aggregate effects of three major policies: job guarantee programs, unemployment insurance, and micro-finance. To find the macroeconomic effects of these policies, we use a general equilibrium occupational choice model that is consistent with salient features of self-employment in developing economies. Our model differs from previous work by introducing unemployment risk, which directly affects the selection of agents into self-employment. Unemployment risk allows the model to generate the mass of *subsistence entrepreneurs* that characterizes developing economies, and shapes the efficacy of economic policy.

The policies we consider have the following effects: Job guarantee programs generate small increases in real wages and limited crowding-out of private employment, as *subsistence entrepreneurs* opt out of self-employment and into the new jobs. Unemployment insurance deters *subsistence entrepreneurship* and increases output, as it increases the size of the most productive enterprises and reallocates labor away from low productivity self-employment. Micro-finance, on the other side, makes self-employment more attractive for individuals with low wealth and productivity, decreasing aggregate productivity as a result.

To illustrate the major features of self-employment in developing countries, we study micro-data from Mexico and India. We highlight three main takeaways. First, there is a

[1] According to the World Development Indicators, 70 percent of the workforce is self-employed in India, 50 percent in Colombia and 30 percent in Mexico. This contrasts with the share of self-employed individuals in developed countries, which tends to be much lower (about 10 percent for Japan, Germany and the U.S.).

U-shaped relationship between self-employment and earnings, with high self-employment rates among the lowest and highest earners of the economy, and most of the self-employed concentrated in the lower end of the earnings distribution.[2] Second, individuals who are more likely to be (income-)constrained are also more likely to opt into self-employment.[3] This is consistent with the presence of *subsistence entrepreneurs*. Third, the self-employed are willing to take on salaried jobs at market wages when such jobs are offered to them. This evidence comes from the implementation of job-guarantee programs,[4] and randomized labor demand shocks to local markets in India documented by Breza, Kaur, and Shamdasani (2017). Importantly, the experimental evidence shows no effect of the increase in labor demand on wages, indicating that self-employment provides slack to the market as low-earning self-employed agents meet the additional demand.

These characteristics are powerful in identifying the relevant mechanisms for the model and distinguishing between model specifications (in the spirit of Nakamura and Steinsson, 2018). In particular, the data rejects the workhorse model frequently used to study entrepreneurship and development (Moll, 2014; Midrigan and Xu, 2014; Buera, Kaboski, and Shin, 2015). This class of models focuses on financial frictions that prevent *productive entrepreneurs* from operating at their optimal size. In these models, self-employment is only taken up by agents who are productive or wealthy enough to earn a higher income in self-employment than as employed workers. Therefore, these models cannot generate the large mass of low-earning self-employed that characterizes developing economies.

We show that a tractable extension of the occupational choice model of Buera, Kaboski, and Shin (2015) that incorporates unemployment risk can account for the patterns in the data. Unemployed agents with low assets cannot tolerate unemployment and turn to self-employment as a source of income regardless of their entrepreneurial ability. This mechanism allows the model to replicate the U-shaped relationship between self-employment and earnings as low-productivity/low-wealth agents opt into self-employment. Crucially, the introduction of unemployment risk also allows the model to match the limited response of wages to labor demand shocks. As in the data, low-earning self-employed agents meet additional labor demand without affecting wages. Absent unemployment risk, the reaction of wages is an order of magnitude higher (as the self-employed have to be compensated to switch occupations), and is strongly at odds

[2] Both developing and developed countries exhibit the U-shaped pattern of self-employment and earnings. However, for developed countries the self-employed are more concentrated among high-earners.

[3] These are unemployed individuals who live in households with no other earners or who do not receive income from other sources (such as remittances).

[4] We study the implementation of the National Rural Employment Guarantee Act (NREGA) in India, a program that provides short-term jobs at market wages in rural districts.

with the experimental evidence.

The model explicitly incorporates relevant dimensions of heterogeneity across agents' occupations (employed, unemployed and self-employed), as well as in their wealth, entrepreneurial ability (productivity), and labor income (if employed). Agents are free to engage in self-employment at any point, but can only become employed following the arrival of a job offer, introducing unemployment risk in a parsimonious way. We lever on recent advances in continuous time methods that give us a computational advantage in the solution of the model (e.g. Moll, 2014; Achdou, Han, Lasry, Lions, and Moll, 2017).

We use a calibrated version of the model to study how self-employment affects the response of the economy to three policies: job guarantee programs, unemployment insurance, and micro-finance.

Job guarantee programs These programs increase labor demand by introducing government provided jobs that involve low skill (typically clerical) tasks and pay (minimum) market wages. The scope of these programs can be massive. For instance, the NREGA program in India reached 53 million beneficiaries in 2010-2011 alone. Because these programs are generally implemented in areas with low (official) unemployment (Feng, Lagakos, and Rauch, 2018), the additional labor demand created by the government is met by crowding out employment from the private sector, or by self-employed agents who take on the new job offers.

We show that when *subsistence entrepreneurs* are prevalent among the self-employed, as the data suggests, the adjustment to the program is carried out by low-earning self-employed individuals. As a consequence, the crowding out of private employment is limited, and the response of wages is low, in line with the data. On the contrary, if self-employment is dominated by *productive entrepreneurs* (as in models without unemployment risk), the response of wages is an order of magnitude higher, as the self-employed must be convinced of leaving their entrepreneurial activities to meet the additional labor demand. In equilibrium, the program increases wages and reduce private demand for labor.

Unemployment insurance We show that unemployment insurance can be used as an instrument to improve productivity, by changing the selection into self-employment, and increasing labor supply. The effects on productivity come in addition to the usual insurance to labor income fluctuations. Even though self-employment is already providing a way for individuals to self-insure against job loss, engaging in self-employment can have negative (long-run) effects on the ability of the individual to regain employment. For instance, recent evidence for the U.S. shows that take up of "gig-economy" jobs (such as Uber or Task-Rabbit) reduces individuals' earnings and employment for at least 4 years (Jackson, 2019).

In our calibrated model, engaging in self-employment reduces the chances of receiving a job offer, causing *subsistence entrepreneurs* to become stuck in self-employment, in turn lowering aggregate productivity.[5]

Unemployment insurance improves productivity by deterring unproductive individuals from engaging in entrepreneurial activities, reducing the share of low-earning self-employed. The implementation of the of the program leads to an increase in output, which is explained by the growth of high-productivity enterprises, and the reduction in the share of *subsistence entrepreneurs*. These effects follow even if the insurance is relatively small. In the simulated exercise we provide insurance equal to 5 percent of the minimum market wage.

Micro-finance These programs are widespread in developing countries, and frequently target poor individuals who lack access to funding for their entrepreneurial activities. These programs can alleviate the financial constraints that curtail the growth of productive entrepreneurs. However, it is well understood that these programs also lower aggregate productivity by inducing low-productivity individuals to engage in entrepreneurship (Buera, Kaboski, and Shin, 2017). The effect of the program over the productivity distribution depends crucially on the initial composition of the self-employed. In models without unemployment risk the change in selection comes from a lower productivity threshold for agents entering into self-employment. This lowers the productivity of the marginal entrant, but still keeps the lowest productivity agents out of self-employment. We show that this mechanism is modified by the introduction of unemployment risk, as poor agents of all productivity levels engage in entrepreneurship *out-of-necessity* (Poschke, 2013a). Therefore, the change in the productivity distribution is driven by the intensity with which the lowest productivity agents opt into self-employment.[6]

Our results suggest that labor market frictions are central to match the characteristics of self-employment in developing economies. We show that models that rely only on financial frictions are at odds with the prevalence of low-earning self-employed in developing countries, as well as the response of wages to labor demand shocks. Furthermore, labor market frictions shape the response of the economy to various policies

[5] This need not be the case *a priori*. The quantitative difference in arrival rates of job offers between unemployed and self-employed agents is implied by the calibration of the model, and informed by lower job-finding rates from self-employment relative to unemployment.

[6] Our results are also consistent with recent experimental evidence on the heterogeneous effects of micro-finance. Banerjee, Breza, Duflo, and Kinnan (2019) show that loans only have a positive lasting effect on individuals who were already engaged in entrepreneurial activities, suggesting that individuals who opt into self-employment due to program have lower productivity, and that productive low-wealth individuals are the ones that benefit from increased access to credit.

by affecting the occupational choice of agents. Unemployment risk drives low-productivity individuals into self-employment, which accounts for the bulk of low-earning self-employed in the data. These *subsistence entrepreneurs* are willing to take on jobs at current market wages, leading to a muted response of wages to labor demand shocks (as in job guarantee programs), in line with experimental and quasi-experimental evidence. They are also sensitive to the provision of unemployment insurance. Offering even low payments to the unemployed drives unproductive individuals out of self-employment, increasing productivity and output in the economy. Finally, the presence of low-earning self-employed is exacerbated by the implementation of micro-finance programs that target the poor. These programs induce more low-productivity poor agents into self-employment, reducing productivity.

Our work is related to a long standing and multifaceted literature on entrepreneurship. Part of the literature focuses on the role of financial constraints that prevent entrepreneurs from operating at their optimal scale (Quadrini, 2000; Cagetti and De Nardi, 2006, 2009). We contribute by highlighting the role of labor market frictions (that induce unemployment risk) in driving self-employment in developing countries. Another part of the literature deals with misallocation and productivity differences across countries (i.a. Hsieh and Klenow, 2009, 2014; Banerjee and Moll, 2010; Restuccia and Rogerson, 2013; Midrigan and Xu, 2014). We contribute to this literature by highlighting the role of selection into entrepreneurship by low-productivity/low-wealth agents facing unemployment risk. These individuals give rise to a mass entrepreneurs who lower aggregate productivity and increase misallocation. We connect this mechanism to policies like unemployment insurance that provide for these agents not to engage in entrepreneurship.

We also relate to papers that highlight alternative mechanisms driving self-employment.[7] Hurst and Pugsley (2016) highlight non-pecuniary benefits as drivers of self-employment in developed economies. Higher transition rates into self-employment for income-constrained agents suggest that non-pecuniary benefits are less relevant for explaining self-employment in developing economies. Hombert, Schoar, Sraer, and Thesmar (2014, 2016) highlight the role of risk in taking on entrepreneurial activities. They show that providing unemployment benefits for individuals exiting from self-employment in France is linked to an increase in entrepreneurship without a decrease in the average quality of new firms. We see our results as complementary, in that we

[7] Our paper is also particularly close to unpublished work by Kevin Donovan, developed independently from ours.

examine the implications of similar policies in a different context: that of a developing economy. Garcia-Cabo and Madera (2019) highlight the role of self-employment as an outside option for workers in the Spanish labor market, characterized by rigid labor contracts. They also document negative selection into self-employment by unemployed agents, and analyze the consequences of self-employment promotion policies targeted at the unemployed.

Finally, our work complements recent papers on the role of structural transformation and wage and unemployment dynamics. During early stages of development a large portion of the population engages in low-productivity activities, akin to those of the *subsistence entrepreneurs* described above. As the country develops workers are pulled from this activities and into more "modern" sectors. Storesletten, Zhao, and Zilibotti (2019) link this pattern to the unresponsiveness of wages to business cycles during early stages of development, studying the case of China. The mechanism is similar to the one described above for job-guarantee programs, with the unproductive sector absorbing workers in downturns, and providing them during booms. It is not until this unproductive sector has shrunk enough that wages are bid up in the development process. Similarly, Feng, Lagakos, and Rauch (2018) document a positive relation between unemployment and GDP per-capita. This suggests that unemployment is an especially poor measure of slack in the labor markets of developing economies, because (in line with our results) self-employment masks the true level of slack of the market (Breza, Kaur, and Shamdasani, 2017).

The rest of the paper is organized as follows: first we describe the main mechanism behind self-employment under unemployment risk. This sets up the discussion of the empirical results that we present in Section 2. We focus on providing a characterization of self-employment with the identification of model specification in mind. Third, we describe the model and its calibration. In Section 4, we show that the model is consistent with the observed features of self-employment. Finally, in Section 5, we use the model to study job guarantee programs, unemployment insurance, and micro-finance.

1 Self-Employment as an outside option

What drives agents to opt into self-employment? Before we describe our empirical evidence we sketch a simple model to highlight the main mechanisms at work. We focus on the forces affecting poor (low-wealth) individuals, who comprise a large share of the population in developing economies. We show that the combination of unemployment risk and limited credit access may induce agents to opt into self-employment, despite having low entrepreneurial ability, generating a mass of low-earning self-employed agents. This mechanism is absent in the presence of strong safety nets (say in form of unemployment benefits),[8] but it is exacerbated by policies like micro-finance. Safety nets mitigate the downside of labor income risk leading to positive selection into self-employment, preventing low-productivity agents from choosing self-employment. In contrast, policies similar to micro-finance reduce the minimum productivity at which agents opt into self-employment.

The model is as follows: an unemployed agent chooses whether to remain unemployed (U) or to become self-employed (S). The agent has a units of assets and a productivity, or entrepreneurial ability, of z. Agents have CRRA utility that depends only on consumption: ($u(c) = {}^{c^{1-\sigma}}/{}_{1-\sigma}$). There is no borrowing or lending.

If the agent chooses to remain unemployed, she will get a job with probability $p \in (0,1]$, becoming employed (E) and receiving a wage $w > 0$. Her consumption will then be $c^E = a + w$. If she does not get a job she will receive unemployment benefits $b \in [0,w)$, and her consumption will be $c^U = a + b$.

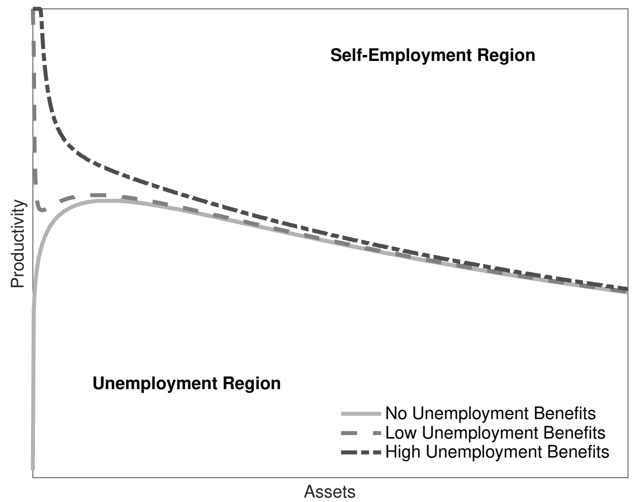
If the agent becomes self-employed she can produce consumption goods using her own assets. Her production depends on her assets and her productivity according to $f(a,z) = za^{\alpha}$, with $\alpha \in [0,1]$. Consumption for the self-employed is: $c^S = a + za^{\alpha}$.

The agent will become self-employed when the utility of doing so exceeds the expected utility of looking for a job. That is, when $u(a + f(a,z)) > p \cdot u(a+w) + (1-p) \cdot u(a+b)$. For each level of wealth a, this inequality defines a threshold value for productivity, above which agents opt into self-employment. Figure 1 shows the threshold value of z for various levels of unemployment benefits (b).

It is instructive to look first at agents as their assets approach zero. The behavior of these agents is heavily influenced by the level of b. This is because as a goes to zero the agent's consumption if unemployed is given by b. When unemployment benefits are low, unemployment becomes intolerable. In that situation, self-employment acts as an outside option bounding the agent's consumption away from zero. This mechanism is strongest when there are no unemployment benefits. In which case, the productivity threshold actually decreases as the agent becomes poorer, even thought the income the agent can generate as a self-employed is decreasing as well. As a consequence, the agent may choose to become self-employed even if she is ill-equipped for the task, lacking the entrepreneurial

[8] This mechanism is also absent under risk sharing, or when there is no labor income risk, features that are common in the macro-development literature, see for instance Buera, Kaboski, and Shin (2015).

Figure 1: Occupational Choice - Unemployment vs Self-Employment



Note: *The figure characterizes the occupational choice of the agent for different levels of unemployment benefits. Lines depict the threshold value of productivity* (z) *for each level of assets* (a) *and a given value of unemployment benefits. The agent chooses self-employment if productivity is above the threshold.*

ability (productivity) or the resources (assets) needed to run a profitable business.[9]

Unlike poor agents, wealthy agents are not sensitive to the level of unemployment benefits. Being able to self-insure against income risk, these agents opt into self-employment only when their entrepreneurial income would be sufficiently high relative to potential labor income. As a consequence there is a positive selection into self-employment with respect to the earning potential of agents, with only high-productivity and high-wealth agents opting into self-employment.[10] The same mechanism is present for poor agents only when they are insured against unemployment.

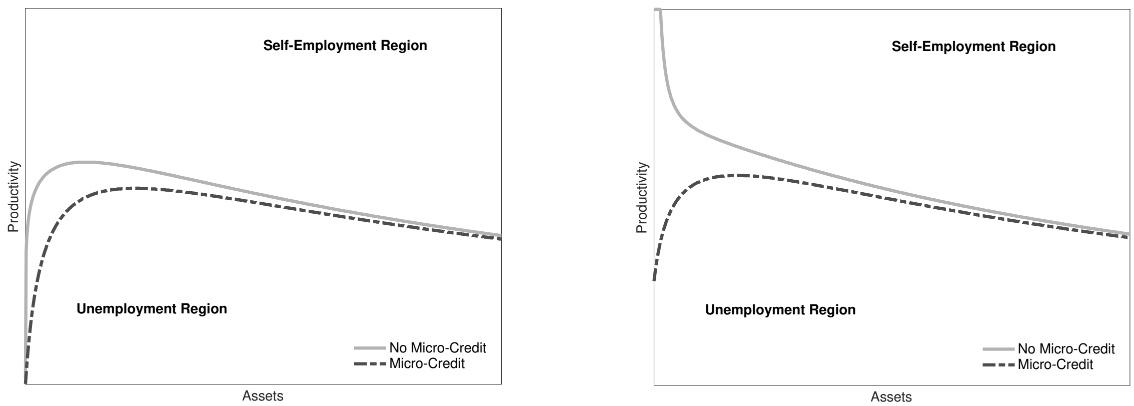
The mechanisms we highlight have important implications for the effects of different policies. In particular for policies targeted to poor agents, like unemployment insurance or micro-finance. As agents opt into self-employment to escape unemployment they form a mass of "*disguised unemployed, or forced entrepreneurs*" (Breza, Kaur, and Shamdasani, 2017). These agents lower overall productivity by diverting resources from the labor market to their own unproductive endeavors. How large this group of agents is depends on the

[9] Paulson and Townsend (2005), elaborate on many of the same ideas developed in this paper on a short policy note. They show evidence from Thailand during the Asian crisis of 1997, after which "*entrepreneurial activity in Thailand increased [...] the number of business households more than doubled*," the authors further note that "*rising unemployment and falling real wages during the crisis led to changes in the types of people who started businesses—and in the types of businesses they started.*"

[10] The selection into self-employment is the same as in Buera, Kaboski, and Shin (2011), where there is no unemployment ($p = 1$), and no uncertainty over wages. The same pattern is shared by many models in the macro-development literature. See for instance Buera, Kaboski, and Shin (2015) and references therein.

Figure 2: Occupational Choice - Unemployment vs Self-Employment with Micro-Credit

(a) No Unemployment Benefits (b) High Unemployment Benefits



Note: *The figure characterizes the occupational choice of the agent for different levels of micro-credit and unemployment benefits. Lines depict the threshold value of productivity* (z) *for each level of assets* (a) *and a given value of micro-credit and unemployment benefits. The agent chooses self-employment if productivity is above the threshold.*

density at the bottom of the wealth distribution, and, crucially, on the access to insurance against income risk and the access to credit markets. As Figure 1 shows, transfers, like unemployment benefits, directly affect the selection into self-employment by preventing unproductive agents from becoming self-employed. Thus, the overall productivity among self-employed agents increases, as does the supply of labor in the economy.[11]

Other policies, like micro-finance, also affect the selection into self-employment. To illustrate this we extend the simple setup described above to allow self-employed agents access to k_{mc} units of seed capital, given to them to jump-start their business. This changes self-employed output to: $f(a, z|k_{mc}) = z(a + k_{mc})^{\alpha}$. Figure 2 shows the new threshold values for productivity with and without unemployment benefits. Despite the simplicity of the setup, it is clear that along with benefiting businesses of productive entrepreneurs, access to micro-finance changes entry into self-employment by making it attractive for less productive agents, potentially reducing allocative efficiency. This mechanism is well known in the literature (see for instance Buera, Kaboski, and Shin (2017)), but there has been little emphasis on the role of unemployment risk in determining the magnitude of the effect on productivity. Absent another form of insuring against unemployment, the access to resources tied to a some form of entrepreneurial activity can lead a considerable mass of agents to opt into self-employment despite having low productivity projects.

[11]The potential of productivity gains from unemployment insurance has been explored before, see for instance Acemoglu and Shimer (1999, 2000). As in our case the gains span from allowing for longer search and better selection.

In Section 3 we extend the setup discussed above into a quantitative occupational choice model with unemployment risk, and discuss more policy implications. But before we discuss the model in detail we turn to empirical evidence characterizing the self-employed in developing economies that provide support for the mechanisms we highlight.

2 Empirical evidence

In this section we aim to characterize some relevant features of self-employment in developing countries. We study both Mexican and Indian data, but we choose to focus our analysis on Mexico.[12] Mexico is in fact representative across developing countries in terms of the level of self-employment and it offers high quality data that allows us to explore the composition of the workforce and transitions in and out of self-employment.

Taken together, our findings are consistent with the prevalence of self-employment out of necessity in Mexico. By this we mean individuals who opt into self-employment when facing low-income spells (such as unemployment) as the ones discussed in the previous Section (see Figure 1). Mexico has a higher self-employment rate than developed countries like the U.S., but the self-employed in Mexico are concentrated among the lowest earning individuals of the country, far from the idea of a productive entrepreneur. The opposite happens in the U.S.. The higher self-employment rate is shaped in particular by a high transition rate from unemployment to self-employment (which also induces a lower unemployment rate for Mexico, relative to the U.S.). These transitions are even higher for agents who are likely to be constrained in terms of access to additional sources of income, indicating that self-employment can be responding to how strong these constraints are.

The features of the self-employed we highlight have direct implications for modeling choices and the effects of development policies. In particular, models of self-employment without unemployment risk or with complete risk sharing are strongly rejected by the data, mainly because of their inability to reproduce the concentration of self-employed among the lowest-earning individuals. These models cannot also reproduce the reaction of wages and workforce composition to job-guarantee programs. We expand on this in Section 4.

[12] Indian data complements our analysis of the Mexican self-employed by allowing us to study the effects of a major job-guarantee program in India, the NREGA. This lets us test whether self-employment is being driven by individuals taking up self-employment out of necessity, rather than by individuals who are particularly attached to, or adept at, their entrepreneurial activities. We find that the implementation of the program is linked to a decrease in self-employment and an increase in unemployment. Consistent with our hypothesis, and experimental evidence showing that self-employment masks the real level of slack in the labor market in developing countries (Breza, Kaur, and Shamdasani, 2017). We present the results in Appendix A.

Labor Status	Our Sample	General Population	U.S.
Worker	68.0%	57.9%	80.7%
Unemployed	2.6%	3.9%	6.3%
Self Employed	29.5%	38.1%	12.9%

Table 1: Workforce Composition

Note: *The data for the Mexican general population is taken from the world development indicators (WDI). The data for the U.S. is taken from the current population survey (CPS).*

Our main analysis is based on the *Encuesta Nacional de Ocupación y Empleo* (ENOE), a household survey administred by the National Institute of Statistics and Geography (INEGI) in Mexico.[13] The ENOE includes a rotating panel in which we observe households for up to 5 quarters. We restrict attention to prime age males (23 to 65 years old) who are head of household and live in one of the ten largest municipalities of Mexico. We cover data from 1995.Q1 to 2015.Q4. In total we study 250 thousands individuals, and have around 1 million observations. We define self-employed as agents who report working in their own (or their family's) business. Table 1 shows the composition of the labor force in our sample, in the whole Mexican economy, and in the U.S. over the period 1995 - 2015. Our sample behaves in a similar way to the overall Mexican labor force, and, we reproduce the differences mentioned above between the U.S. and Mexico in terms of self-employment and unemployment.

Our findings match previous studies on Mexican entrepreneurship, which highlight that "[a] *high level of self-employment, combined with the predominance of micro-enterprises, is a distinctive feature of entrepreneurship in Mexico*" (OECD, 2012). In fact, Mexico has an enterprise birth-rate that doubles that of the USA, a self-employment rate of around 35%, and more than 90% of companies have less than 9 employees, compared to 60% in the U.S..

We now turn to characterize transitions in the Mexican labor market. Table 2 reports the average quarterly transition rates between employed, unemployed and self-employed individuals. The high self-employment rate reported in Table 1 is explained by high transitions rates into self-employment (8.1 percent of the employed and 26.9 percent of the unemployed transition to self-employment in a given quarter). On the other hand, the low unemployment rate is explained by low transitions into unemployment, with rates in the order or two percent, coupled with high transitions out of unemployment.[14] In

[13] Further information in http://en.www.inegi.org.mx/proyectos/enchogares/historicas/enoe/.

[14] At a quarterly frequency we are unable to observe short term unemployment spells. Crucially for us, these include not only workers who are rehired within the same quarter, but also workers who switch to self-employment after loosing their job, inflating the transition rate from employment to self-employment and deflating the transition from unemployment to self-employment. Regardless, the behavior of the latter

	Worker	Unemployed	Self-Employment
Worker	90.2%	1.7%	8.1%
Unemployed	47.1%	26.7%	26.9%
Self-Employment	19.2%	2.0%	79.0%

Table 2: Quarterly Transition Rates

Note: *The figure characterizes the occupational choice of the agent for different levels of unemployment benefits. Lines depict the threshold value of productivity (z) for each level of assets (a) and a given value of unemployment benefits. The agent chooses self-employment if productivity is above the threshold.*

Appendix D.1 we provide more evidence showing that the high transition rates from unemployment to self-employment are not due to observable differences between individuals. To do so, we follow the same strategy as Katz and Krueger (2017). We find that the transition rates of unemployed agents to self-employment are 20.9 percentage points higher than those exhibited by comparable employed individuals (Table D.1).[15]

The observed composition of the labor market (high self-employment coupled with low unemployment), and the transition rates in the Mexican labor market are both consistent with unemployment being intolerable for individuals. Yet, it is still unclear what type of self-employed is most prevalent in Mexico, and what forces determine the high transition rates into self-employment. In the following two sections we tackle these questions. Section 2.1 further characterizes the type of self-employed individuals who are present in Mexico. We show that the distribution of the self-employed in Mexico is concentrated among low-earning individuals, as opposed to being concentrated among high-earners as in the U.S., indicating the prevalence of low-productivity self-employed. In Section 2.2, we ask whether more constrained individuals transition more into self-employment. This informs of about the mechanisms laid out in Section 1. We do in fact find that individuals who have less access to outside sources of income transition more to self-employment.

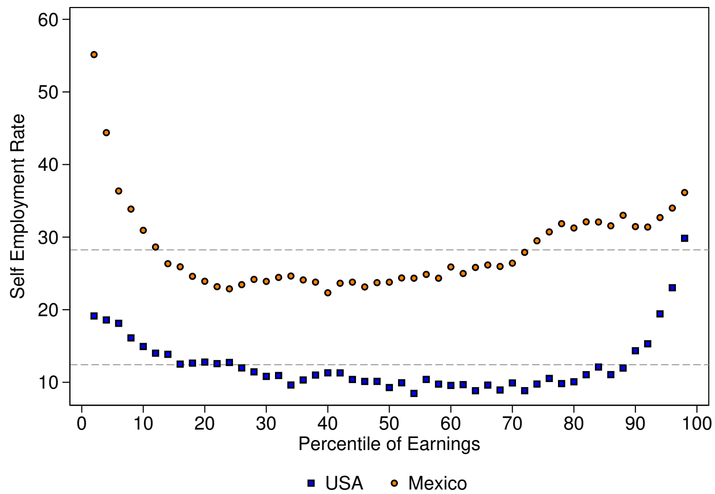
2.1 Self-Employment is concentrated among low earners

We first characterize the prevalence of different forms of self-employment across developing and developed countries. The main hurdle in describing the self-employed is that they encompass agents engaging in different economic activities. Most notably, some

type of switchers is consistent with the idea of self-employment out of necessity (as discussed in Section 1).

[15] Our results align with those of Katz and Krueger (2017) for the U.S.. They find that unemployed individuals are more likely to transition to an alternative work arrangement job than agents who are employed. Alternative work arrangements (e.g. working for Uber or Task Rabbit) have a similar role as self-employment in Mexico, namely offering a self-procured source of income.

Figure 3: Self-Employment rate by percentile of the earnings distribution



Note: *The figure reports the share of the population classified as self-employed for bins of the earnings distribution. Each bin corresponds to three percent of the population. The blue squares correspond to U.S. data from the Current Population Survey (CPS). The orange circles correspond to Mexican data from the ENOE. The horizontal dashed lines correspond to the average self-employment rate in each country.*

self-employed are productive entrepreneurs who own (and sometimes manage) large scale firms, some operate small scale businesses (e.g., mom-and-pop shops), while others engage in unproductive endeavours that only provide subsistence income. To surpass this hurdle we examine the share of self-employed agents by percentiles of the earnings distribution for Mexico and the U.S.. This approach allows us to get a snapshot of the concentration of self-employed of various types without having to adhere to any predetermined grouping. Figure 3 shows the results.[16]

Self-employment is more prevalent in the tails of the earnings distribution, with both countries exhibiting a U-shaped relation between the self-employment rate and earnings despite differences in levels (see Table 1). Yet, the relation differs across countries. Self-employment is more salient for individuals at the bottom of the earnings distribution for Mexico than for the U.S., while the opposite thing happens at the upper end of the distribution, where the increase in the self-employment rate is more marked in the U.S. than in Mexico.

The difference in the relation between self-employment and earnings is consistent with

[16]To compute Figure 3 we first run a regression of the form $\log(earn_{i,t}) = \alpha + \gamma_t + \beta X_{i,t} + \eta_{i,t}$, where $earn_{i,t}$ corresponds to the earnings of individual i at time t, and X is a vector of individual level controls. We rank $\hat{\eta}_{i,t}$ and classify them in bins of 3 percent of the sample, and then compute the self-employment rate in each of these bins. The pattern we report is robust when we use raw earnings instead of controlling for observables. We use data from the Current Population Survey for the U.S.

the mechanisms sketched in Section 1. The lack of a safety net, coupled with labor market frictions, can drive low productivity agents into self-employment as they search for viable income options, inducing a high self-employment rate among those at the bottom of the earnings distribution. This contrasts with the implications of having limited access to capital (due to dysfunctional financial markets or information frictions), which can help explain why self-employed agents have lower earnings in developing countries, but are by themselves at odds with the higher level of self-employment and its concentration at the bottom of the distribution. In fact, we show in Section 4 that a combination of labor market and capital market frictions are required to generate the U-shaped relation between self-employment and earnings.

2.2 Constrained agents transition more into self-employment

In this Section we investigate further the potential reasons for an individual to opt into self-employment. We ask whether individuals who are more constrained (in terms of earnings) are more likely to become self-employed. To answer this we focus on unemployed individuals, both because they are more likely to be constrained than employed agents (who have an income), and because most of the transition into self-employment in Mexico comes from the unemployed (see Tables 2 and D.1).

To be clear, there are other forces that affect the decision to become self-employed. For instance, individuals can have preferences for self-employment,[17] or there might be differences across individuals in their (perceived) entrepreneurial ability.[18] Nevertheless, we focus on the empirical relevance of individuals opting into self-employment out of necessity (i.e. as an an outside option for individuals experiencing an unemployment spell). There are two main reasons for this, first, as we have explained earlier, if self-employment is taken out of necessity, the criteria for which type of individual opts into self-employment depends less on the agent's entrepreneurial ability and more on her access to forms of income outside of the labor market. This in turn changes the distribution of productivity across the self-employed, inducing a larger share of unproductive small enterprises, and reducing aggregate productivity. Second, the decision to become self-employed out of necessity can

[17] This was proposed by Hurst and Pugsley (2016) to explain the patterns of self-employment in the U.S.. Self-employment gives access to independence at work, control over work schedules and self-determination of tasks. An individual can then choose to be self-employed even if doing so decreases her income due to non-pecuniary rewards.

[18] Higher ability would increase an individual's expected earnings (relative to a salaried job) and make self-employment more attractive. Equivalently, the individual can have traits that make her have low prospects in the labor market (e.g. a low job-finding rate), inducing her to opt into self-employment.

be influenced by policy more directly than the preferences or ability of the population, as the example in Section 1 makes clear.

To test whether more constrained individuals transition more into self-employment we use two variables to proxy for access to additional sources of income: the presence of a dual-earner (as in Chetty (2008)), and the reception of remittances from relatives living abroad.[19] We then estimate the effect of being less constrained on the transitions from unemployment. Tables 3 and 4 show the results. A consistent message emerges. Individuals with additional sources of income have lower transition rates to self-employment. We interpret these results as indicative that constrained agents do in fact transition more into self-employment.[20]

Moreover, we also observe that less constrained individuals transition more out of self-employment. See the coefficients on the interaction between being self-employed and having a second earner in Tables D.2 and D.3. Self-employed agents are on average 2.4 percentage points more likely to transition to employment if they have a second earner. This is precisely the relation we would expect if self-employment out of necessity is prevalent. Less constrained self-employed can in principle devote more time/effort to search activities, and transition to a salaried job for which they might be better suited.

Turning to the results in Table 3, we estimate that individuals with a second earner have probabilities of transition to employment that are 3.2 percentage points higher and transitions to self-employment that are 3.9 percentage points lower (that is a 17% decrease, from 22.2 to 18.3) with respect to individuals without a second order. We also test if the individual's (self-reported) actions towards job search change with the presence of a second earner. Table D.4 in Appendix D presents the results. We find that most job-search activities (e.g. examining job postings, looking for temporary jobs) are not significantly different for individuals with a second earner. Importantly there is no difference on whether agents report being making plans to start their own business. The only significant differences are that workers with a second earner are, on average, 1.5 years older, and (presumably as a consequence) are less likely to use the internet to find a job.

Table 4 presents the results using remittances as a proxy for the resources of the individual. Individuals who receive remittances in times of unemployment transition at lower rates to self-employment. The coefficient, -8 percentage points, is economically

[19] We thank Sylvain Catherine who suggested the latter exercise.

[20] The results can also be interpreted as an indication of a limited role for preferences and ability for self-employment as main drivers of the transitions to self-employment. If they were, we should observe people who have external sources of income being more prone to become self-employed, as they could enjoy the non-pecuniary benefits of work independence, or try their luck at entrepreneurship, while not worrying (as much) about low income levels.

	(1) U→E	(2) U→S	(3) U→U	(4) U→I
Second Earner	0.032***	-0.039**	0.007	-0.000
	[0.010]	[0.018]	[0.015]	[0.000]
Age	-0.008***	0.003***	0.005***	0.000
	[0.000]	[0.000]	[0.000]	[0.000]
Constant	0.835***	0.209	-0.044	-0.001
	[0.301]	[0.326]	[0.098]	[0.002]
Observations	8376	8376	8376	8376
Mean Dep. Variable	0.505	0.222	0.272	0.000104
Schooling Controls	Yes	Yes	Yes	Yes
Time Fixed Effect	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes

Table 3: Second Earner and Transitions from Unemployment

Note: *The LHS variable is an indicator variable that takes the value of one if the individual experienced the transition specified in each column. U denotes unemployment, E salaried work, S self-employment, and I inactivity. Second Earner is an indicator variable that takes the value of one if the individual's couple was an income earner in period $t-1$. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. The sample consists of individuals who were unemployed in period $t-1$. The regressions are run by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

	(1) U→E	(2) U→S	(3) U→U	(4) U→I	(5) U→S
Remittances	0.058	-0.080***	-0.033	0.055	
	[0.053]	[0.021]	[0.040]	[0.037]	
Age	-0.012***	0.002***	0.002**	0.008***	0.001***
	[0.000]	[0.000]	[0.001]	[0.001]	[0.000]
Latent Remittances					-0.045
					[0.036]
Constant	1.237***	0.147	-0.168***	-0.216	0.177
	[0.262]	[0.202]	[0.050]	[0.227]	[0.114]
Observations	8615	8615	8615	8615	25135
Mean Dep. Variable	0.463	0.188	0.256	0.0932	0.188
Schooling Controls	Yes	Yes	Yes	Yes	Yes
Time Fixed Effect	Yes	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

Table 4: Remittances and Transitions from Unemployment

Note: *The LHS variable is an indicator variable that takes the value of one if the individual experienced the transition specified in each column. U denotes unemployment, E salaried work, S self-employment, and I inactivity. Remittances is an indicator variable that takes the value of one if the individual reported having received remittances in period $t-1$. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. The sample consists of individuals who were unemployed in period $t-1$. The regressions are run by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

significant, considering mean transition rates are 18.8 percentage points in this sample. Moreover, we would expect the coefficients in this regression to have an upward bias. It is reasonable to expect that people who need financial help abroad are also those who have lower probabilities of finding a good job, therefore creating an spurious positive coefficient in column (2) and a negative bias in column (1). The fact that we observe the opposite (people who receive remittances transition less to self-employment) is reassuring. Of course, we cannot rule other sources of bias since the reception of remittances is not created by exogenous variation across individuals.

As we did for Second Earners, we also check whether receiving remittances has an effect on search activities, or the intent to start a business. Table D.5 in Appendix D presents the results. Receiving remittances has a weak effect overall, it is associated with a lower likelihood of using the internet to look for jobs, and a lower likelihood of of searching for temporary employment, however it is also associated with a higher likelihood of asking directly for a job.

3 Model

In this section we describe a quantitative occupational choice model where agents face unemployment risk. The objective of the model is to illustrate the role of self-employment in shaping the response to various policies in developing economies. We extend the baseline macro-development model in Buera, Kaboski, and Shin (2015) of employed (E) and self-employed (S) agents by introducing unemployment (U) and allowing for transitions between occupations. Occupations differ with respect to their income source and to whether or not agents can freely opt into them. With the aim of being parsimonious while capturing the major forces at play, we assume that agents can freely move into unemployment and self-employment, while they can only become employed following an exogenous job offer, which we interpret as a stand-in for underlying search frictions in the labor market. We calibrate the model to match salient features of the Mexican labor market, being consistent with the evidence we presented in Section 2. In Section 5 we use the model to conduct counterfactual experiments designed to evaluate three major policies that affect the occupational choice of agents.

The model is of a small open economy populated by a continuum of agents. Time is continuous and goes forever. Agents are heterogeneous with respect to their occupation $\{E, U, S\}$, and with respect to their labor efficiency (ϵ), productivity (z), and asset holdings (a). If employed, an agent's labor income is given by $w\epsilon$, where w is the economy-wide wage rate per efficiency unit of labor. Unemployed agents have zero labor efficiency and receive a constant income b.[21] If self-employed, an agent produces final goods using capital and labor with a technology indexed by the agent's productivity (which we also refer to as the agent's entrepreneurial ability). The agent's income is given by the profits of operating her technology.

As is standard in the literature, there is limited access to credit markets. Employed and unemployed agents face a borrowing limit $\underline{a} \leq 0$. Self-employed agents can borrow to obtain productive capital, but they face a borrowing constraint that depends on their assets (which are used as collateral): $k \leq \lambda a$. These borrowing constraints capture information frictions or commitment problems, which we do not model explicitly. See, among others, Cagetti and De Nardi (2006) and Buera et al. (2011) for micro foundations of the borrowing constraint.

As we mentioned earlier, any agent can become self-employed or unemployed at will. In contrast, transitions to employment are governed by an exogenous process that captures (in a reduced form) the arrival of job offers. In particular, we assume that an agent's labor efficiency is set to zero as long as they are not employed. The agent gets to draw a positive labor efficiency value following the arrival of an exogenous job offer, which follows a Poisson process with arrival rate γ^o, that depends in turn on the agent's occupation $o \in \{U, S\}$. Once the agent draws a new value for her labor efficiency, she gets to accept it and become employed, or reject it, maintaining her previous occupation. Finally, employed agents are subject to job destruction shocks with arrival rate γ^E.

We aim to keep our modeling of the economy as simple as possible, and as close as possible to he standard modeling of entrepreneurship in the macro-development literature (e.g., Buera, Kaboski, and Shin, 2015). Yet, we depart from previous models in two important and intertwined features. First, we include labor income risk by letting the labor efficiency of employed agents fluctuate. Second, we prevent agents to have access to employment at will by including an unemployment state with no labor income.[22] Without these features employment acts as an outside option from self-employment (where income is volatile, following changes in productivity). Consequently, the model is unable to produce the type of low-earning self-employed agent that characterizes self-employment in

[21] It is possible to think about income while unemployed as coming from home production, or from transfers from family members or government agencies. In Section 5 we take the latter view and examine what happens if unemployed income b increases.

[22] Most of the literature assumes labor income to be common to all agents (given by a market wage), and allows agents to opt into the employment state in a frictionless manner, instead introducing costs to engage in entrepreneurial activities.

developing economies. In contrast, self-employment acts in our model as an outside option for agents facing low-income (unemployment) spells, as well as for agents with high entrepreneurial ability. This allows to capture the joint distribution of self-employment and earnings, as seen in Figure 3. Moreover, the behavior of agents opting into self-employment plays an important role in the evaluation of various policies, as we already hinted in 1. We expand on the fit of the model and its policy implications in Sections 4 and 5.

We solve for the stationary equilibrium of the model. Appendix B presents a version of the baseline model without unemployment. This alternative model is a useful point of reference to judge the performance of the model, as we show in Section 4. Appendix C discusses the computational implementation of the model's solution.

3.1 Stochastic Processes

We assume that labor efficiency, ϵ, and productivity, z, follow independent Poisson processes with arrival rates γ^{ϵ} and γ^{z} respectively. Upon arrival the agent draws a new value for the state (either ϵ or z) from a conditional probability distribution $\Pr^{\epsilon}(\epsilon'|\epsilon)$ or $\Pr^{z}(z'|z)$. When an unemployed or a self-employed agent receives a job offer, events with Poisson arrival rates γ^{U} and γ^{S}, she draws a value of labor efficiency from $\Pr^{U}(\epsilon)$ or $\Pr^{S}(\epsilon)$ respectively.

3.2 Agent's Problem

The problem of an agent depends on her occupation. We discuss each occupations in turn. In what follows we denote as V^{o} the value of an agent in occupation $o \in \{E, U, S\}$, taking into account the occupational choice of the agent, and we denote as $\tilde{V}^{o}$ the value that the agent would receive if she were to remain in occupation o.

Employed agents The employed receive an income of $w\epsilon$ and are subject to job destruction shocks that arrive at a rate γ^{E}. If the shock arrives the agent becomes unemployed. The agent can also choose to become self-employed at any instant. The value for an employed agent that remains employed is given by $\tilde{V}^E$:

$$\begin{aligned}\rho\tilde{V}^{E}\left(a,z,\epsilon\right) &= \max_{c}\, u\left(c\right)+V_{a}^{E}\left(a,z,\epsilon\right)\dot{a}+\gamma^{E}\left(V^{U}\left(a,z\right)-\tilde{V}^{E}\left(a,z,\epsilon\right)\right) \\ &\quad +\gamma^{z}\int\left(V^{E}\left(a,z',\epsilon\right)-\tilde{V}^{E}\left(a,z,\epsilon\right)\right)d\mathrm{Pr}^{z}\left(z'|z\right) \\ &\quad +\gamma^{\epsilon}\int\left(V^{E}\left(a,z,\epsilon'\right)-\tilde{V}^{E}\left(a,z,\epsilon\right)\right)d\mathrm{Pr}^{\epsilon}\left(\epsilon'|\epsilon\right) \\ \text{s.t.} &\qquad \dot{a}=w\epsilon+ra-c \qquad a\geq\underline{a}\end{aligned} \tag{1}$$

Unemployed agents The unemployed receive an income of b and are subject to job offers that arrive at a rate γ^U, they are free to reject an offer depending on their current assets and productivity, and the labor efficiency they would have if employed. The agent can also choose to become self-employed. The value for an agent that remains unemployed is given by $\tilde{V}^U$:

$$\begin{aligned}\rho\tilde{V}^{U}\left(a,z\right) &= \max_{c}\, u\left(c\right)+V_{a}^{U}\left(a,z\right)\dot{a} \\ &\quad +\gamma^{U}\int\max\left\{V^{E}\left(a,z,\epsilon\right)-\tilde{V}^{U}\left(a,z\right),0\right\}d\mathrm{Pr}^{U}\left(\epsilon\right) \\ &\quad +\gamma^{z}\int\left(V^{U}\left(a,z'\right)-\tilde{V}^{U}\left(a,z\right)\right)d\mathrm{Pr}^{z}\left(z'|z\right) \\ \text{s.t.} &\qquad \dot{a}=b+ra-c \qquad a\geq\underline{a}\end{aligned} \tag{2}$$

Self-employed agents The self-employed agent receive income from profits, $\pi\left(a,z\right)$, generated by their productive activities. Self-employed agents receive job offers at a rate γ^S. Upon arrival of an offer the agent is free to reject it. The agent can also choose to become unemployed. The value for an agent that continues being self-employed is $\tilde{V}^S$:

$$\begin{aligned}\rho\tilde{V}^{S}\left(a,z\right) &= \max_{c}\, u\left(c\right)+V_{a}^{S}\left(a,z\right)\dot{a} \\ &\quad +\gamma^{S}\int\max\left\{V^{E}\left(a,z,\epsilon\right)-\tilde{V}^{S}\left(a,z\right),0\right\}d\mathrm{Pr}^{S}\left(\epsilon\right). \\ &\quad +\gamma^{z}\int\left(V^{S}\left(a,z'\right)-\tilde{V}^{S}\left(a,z\right)\right)d\mathrm{Pr}^{z}\left(z'|z\right) \\ \text{s.t.} &\qquad \dot{a}=\pi\left(a,z\right)+ra-c \qquad a\geq\underline{a}\end{aligned} \tag{3}$$

The optimal consumption decision can be found in all cases from the first order condition of the agent's problem (see Achdou et al. (2017)). Letting $o\in\{E,U,S\}$ denote the occupational state of the agent, we have:

$$c = u'^{-1} \left(V_a^o \right) . \tag{4}$$

It is only left to account for occupational choice of the agents. At every instant the value of an agent must reflect the upper envelope of the choices available to her. This works akin to a value matching condition in optimal stopping time problems (Stokey, 2009). The following conditions must hold:

$$V^E (a, z, \epsilon) = \max \left\{ \tilde{V}^E (a, z, \epsilon) , \tilde{V}^U (a, z) , \tilde{V}^S (a, z) \right\} \tag{5}$$

$$V^U (a, z) = \max \left\{ \tilde{V}^U (a, z) , \tilde{V}^S (a, z) \right\} \tag{6}$$

$$V^S (a, z) = \max \left\{ \tilde{V}^U (a, z) , \tilde{V}^S (a, z) \right\} \tag{7}$$

The last two conditions imply that unemployment and self-employment must give the same value to the agent, this is because the agent can instantaneously move between the two occupations, so any difference in value is in a sense arbitraged away.

For future reference let $\chi^{oo'}$ be an indicator function for the occupational choice of the agents. Then for the employed we have

$$\chi^{Eo'} (a, z, \epsilon) = \begin{cases} 1 & \text{if } V^E (a, z, \epsilon) = \tilde{V}^{o'} (a, z) \\ 0 & \text{otherwise} \end{cases} ,$$

where $o' \in \{U, S\}$. For the unemployed and the self-employed we have:

$$\chi^{US} (a, z) = \begin{cases} 1 & \text{if } V^U (a, z) = \tilde{V}^S (a, z) \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \chi^{SU} (a, z) = \begin{cases} 1 & \text{if } V^S (a, z) = \tilde{V}^U (a, z) \\ 0 & \text{otherwise} \end{cases} .$$

Of course $\chi^{oo} = 1$ indicates no change in the agent's occupation.

3.3 Self-Employed production technology

The profits of a self-employed agent are given by:

$$\pi (a, z) = \max_{k \leq \lambda a \, , \, n \geq 0} \left\{ z \left(k^{\alpha} n^{1-\alpha} \right)^{\nu} - wn - (r + \delta) k \right\} , \tag{8}$$

where $\alpha \in (0,1)$ and $\nu \leq 1$. The The solution to the profit maximization problem when $\nu < 1$ is:

$$n(a,z) = \left(\frac{\nu(1-\alpha)z}{w}\right)^{\frac{1}{1-(1-\alpha)\nu}} (k(a,z))^{\frac{\alpha\nu}{1-(1-\alpha)\nu}} \ , \tag{9}$$

with capital demand given by:

$$k(a,z) = \min\left\{\nu^{\frac{1}{1-\nu}} z^{\frac{1}{1-\nu}} \left(\frac{\alpha}{r+\delta}\right)^{\frac{1-(1-\alpha)\nu}{1-\nu}} \left(\frac{1-\alpha}{w}\right)^{\frac{(1-\alpha)\nu}{1-\nu}}, \lambda a\right\} . \tag{10}$$

If $\nu = 1$ the solution is:

$$n(a,z) = \left(\frac{(1-\alpha)z}{w}\right)^{\frac{1}{\alpha}} k(a,z) \ , \text{ with } k(a,z) = \lambda a \mathbf{1}_{\{z \geq \underline{z}\}} . \tag{11}$$

The threshold $\underline{z} = \left(\frac{r+\delta}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha}$ is the minimum value of z for which there is production.

3.4 Labor market

The labor market is formally frictionless with efficiency units of labor being transacted at a wage-rate w. We assume that unemployed and self-employed agents have no efficiency units of labor to transact (i.e. $\epsilon = 0$), and only get them following the arrival of a "job offer." Total labor supply (in efficiency units) is then given by the integral over the efficiency units of employed agents:

$$N^S = \int \epsilon dG^E \ , \tag{12}$$

where G^E is the distribution of employed agents in the economy.

The only source of labor demand is the businesses of the self-employed. Total labor demand is thus given by:

$$N^D = \int n(a,z) \, dG^S \, , \tag{13}$$

where G^S is the distribution of self-employed agents in the economy.

It is convenient to sketch the main mechanisms at work in the labor market for the discussions in Sections 4 and 5. Consider then an increase in the wage rate. First, the market responds statically by decreasing labor demand, as existing firms decrease their labor demand. This effect is accompanied by an increase in the number of self-employed agents who accept job offers, as their self-employed income goes down and their employed income increases. The change in occupational choice further decreases labor demand while also increasing labor supply. Provided that the job offer arrival rate is higher for unemployed agents $\left(\gamma^U > \gamma^S\right)$ there is an additional effect, as some self-employed agents opt into unemployment to increases their chances of a salaried job. The net effect is, as expected, a decrease in the net demand for labor.

3.5 Equilibrium

An stationary equilibrium for this economy is a set of value functions $\left\{V^o, \tilde{V}^o\right\}_{o\in\{E,U,S\}}$, along with an optimal consumption function $\{c^o\}_{o\in\{E,U,S\}}$, labor and capital demand from self-employed $\{n, k\}$, prices $\{r, w\}$ and a distribution of agents for each occupation $\{G^o\}_{o\in\{E,U,S\}}$, such that:

1. Value functions are consistent with the agent's optimization. That is, they satisfy equations (1)-(3) and equations (5)-(7).
2. Consumption (and thus asset accumulation) are consistent with the agent's optimization. That is, it is given by equation (4).
3. Capital and labor demand solve the self-employed's profit maximization problem. That is, they are given by (9) and (10) if $\nu < 1$, or by (11) if $\nu = 1$.
4. Labor market clears: $N^S = N^D$, where total labor supply is given by (12), and total labor demand by (13).
5. The interest rate is given by the international interest rate $r^\star$.
6. The distribution of agents is stationary. For this the densities g^E, g^U and g^S of employed, unemployed and self-employed agents must satisfy the system of Kolmogorov Forward Equations (KFE) defined by:[23]

$$\begin{aligned} 0 = &-\frac{\partial}{\partial a}\left[\dot{a}g^U(a,z)\right] + \int \chi^{EU}(a,z,\epsilon)\, g^E(a,z,\epsilon)\, d\epsilon + \chi^{SU}(a,z)\, g^S(a,z) \\ &- \chi^{US}(a,z)\, g^U(a,z) - \gamma^U \mathrm{Pr}^U(\epsilon)\, g^U(a,z)\, \mathbb{1}_{\{V^E(a,z,\epsilon)>V^U(a,z)\}} \\ &- \gamma^z \int \mathrm{Pr}^z(z'|z)\, g^U(a,z)\, dz' + \gamma^z \int \mathrm{Pr}^z\left(z|z'\right) g^U(a,z')\, dz' \end{aligned} \tag{14}$$

[23] The density functions are also such that: $1 = \int\int\left(\int g^E(a,z,\epsilon)\, d\epsilon + g^U(a,z) + g^S(a,z)\right) da\, dz$. So that $\int g^E(a,z,\epsilon)\, da\, dz\, d\epsilon$ gives the mass of employed agents, and $\int g^o(a,z)\, da\, dz$ for $o \in \{U,S\}$ gives the mass of unemployed and self-employed agents.

$$0 = -\frac{\partial}{\partial a}\left[\dot{a}g^{S}(a,z)\right] + \int \chi^{ES}(a,z,\epsilon)\, g^{E}(a,z,\epsilon)\, d\epsilon + \chi^{US}(a,z)\, g^{U}(a,z) \tag{15}$$
$$- \chi^{SU}(a,z)\, g^{S}(a,z) - \gamma^{S}\mathrm{Pr}^{S}(\epsilon)\, g^{S}(a,z)\, \mathbb{1}_{\{V^{E}(a,z,\epsilon)>V^{S}(a,z)\}}$$
$$- \gamma^{z}\int \mathrm{Pr}^{z}(z'|z)\, g^{S}(a,z)\, dz' + \gamma^{z}\int \mathrm{Pr}^{z}\left(z|z'\right) g^{S}(a,z')\, dz'$$

$$0 = -\frac{\partial}{\partial a}\left[\dot{a}g^{E}(a,z,\epsilon)\right] - \gamma^{E} - \chi^{EU}(a,z,\epsilon)\, g^{E}(a,z,\epsilon) - \chi^{ES}(a,z,\epsilon)\, g^{E}(a,z,\epsilon) \tag{16}$$
$$+ \gamma^{U}\mathrm{Pr}^{U}(\epsilon)\, g^{U}(a,z)\, \mathbb{1}_{\{V^{E}(a,z,\epsilon)>V^{U}(a,z)\}}$$
$$+ \gamma^{S}\mathrm{Pr}^{S}(\epsilon)\, g^{S}(a,z)\, \mathbb{1}_{\{V^{E}(a,z,\epsilon)>V^{S}(a,z)\}}$$
$$- \gamma^{\epsilon}\int \mathrm{Pr}^{\epsilon}(\epsilon'|\epsilon)\, g^{E}(a,z,\epsilon)\, d\epsilon' + \gamma^{\epsilon}\int \mathrm{Pr}^{\epsilon}\left(\epsilon|\epsilon'\right) g^{E}(a,z,\epsilon')\, d\epsilon'$$
$$- \gamma^{z}\int \mathrm{Pr}^{z}(z'|z)\, g^{E}(a,z,\epsilon)\, dz' + \gamma^{z}\int \mathrm{Pr}^{z}\left(z|z'\right) g^{E}(a,z',\epsilon)\, dz'$$

3.6 Discussion of modeling assumptions

We have already mentioned the rationale behind some of our modeling choices, but it is worthwhile to further discuss the assumptions we place on the model before moving forward to the quantitative implications of the model. In particular, we want to discuss two key assumptions placed on the functioning of the labor market. The first one is that labor demand comes exclusively from the self-employed. This is clearly not the case in reality, and it imposes a tight link between labor demand and labor supply. In the model, increases in the labor supply require reassigning workers to employment, and that reduces the number of self-employed, reducing in turn labor demand. Because of this link, equilibrium wages become very responsive to changes in the occupational choices of agents (as we see in Sections 5.2 and 5.3). This link can be weakened by introducing a corporate sector that offers an additional source of labor demand (as in Kitao (2008)), or by separating the labor demand completely from the problem of the self-employed, instead having a new sector that mixes entrepreneurial output with labor to produce final goods (as in Guvenen, Kambourov, Kuruscu, Ocampo, and Chen (2019)). We choose not to pursue these extensions because our objective with the model is to highlight the mechanisms behind the mass of low-productivity self-employed that characterizes developing economies. In that order, we we choose to keep the model as simple as possible.

The second assumption is that there is no endogenous response of job finding rates (γ^U, γ^S) to changes in the number of job searchers, or the productivity (scale) of firms. As we mentioned above, we choose this modeling of labor market frictions aiming for parsimony and comparability with other models used in the macro-development literature. This assumption also simplifies the computational burden of the model.[24] Of course, this assumption does not come without consequences, the main of which is the muting of general equilibrium effects that can affect the response to policy changes.[25] Nevertheless, these general equilibrium effects are likely to strengthen the results we provide for the three policies we consider. We further discuss this issue in Section 5.

Additionally, there are two assumptions on the problem of the self-employed that we also wish to discuss. The first one is that the self-employed have access to capital at the global (risk-free) interest rate. This interest rate is lower than the rate at which entrepreneurs in developing countries actually face. As a consequence, self-employment in the model becomes more attractive and the optimal scale of businesses increases. Even though this tends to generate larger businesses, the quantitative effect of the lower interest rate is small for the least productive self-employed. The optimal scale at the bottom of the productivity distribution is close to zero in any case. Because we are mostly concerned by capturing the mass of low-productivity self-employed we don't consider this assumption to be too stringent on the model.

Finally, we assume that all the self-employed operate the same technology differing only in productivity, and that there are no installation costs for agents to become self-employed. Installation costs are generally understood to vary with the type of technology being adopted by the entrepreneur, as in Midrigan and Xu (2014) or Buera et al. (2011), with *better* technologies having higher installation costs. In this case, the low-asset/low-productivity agents are likely to opt for the *worse* technology, which, we argue, should have close to zero cost of adoption. Therefore, introducing a menu of technologies and installation costs would only strengthen our results by making low-productivity self-employment more attractive for low-wealth agents. Because of this, we choose to abstract from this margin, keeping the model simple while still being able to

[24] We see the computational advantages of the model as more than a mere convenience. The way in which we pose the model allows practitioners to use standard tools from dynamic programming to solve for the model in general equilibrium.

[25] For instance, as we show in Section 5.2, if there is an increase in unemployment benefits the number of unemployed workers increases, reducing labor demand. We do not take into account the effect of this increase on market tightness, which would spur vacancy creation among firms still in operation. Absent these effects all the adjustment in the labor market has to come through the wage rate.

capture the presence of low-productivity entrepreneurs.

4 Quantitative analysis

We now turn to determine the ability of the model to match salient features of self-employment in developing economies. As we show below, the model is successful in most dimensions. Further, the data strongly rejects an alternative version of the model without unemployment risk (Appendix B), while our baseline model is shown to be compatible with the empirical evidence presented in Section 2 and Appendix A.

To match the data we calibrate the model by choosing parameters in two ways. A first group of parameters is externally calibrated, with values taken from the literature or chosen independently of the equilibrium outcomes of the model. A second group of parameters is chosen to match targeted moments of the earnings distribution, workforce composition, and transition rates across occupations from the Mexican data discussed in Section 2. Before discussing the parameter values, we define the parameterization of the stochastic processes governing productivity and labor efficiency. After this is done we evaluate the model's performance in terms of matching targeted and untargeted moments of the data.

Stochastic processes and discretization of states We discretize the processes for productivity (z) and labor efficiency (ϵ) so that the conditional probability distributions $\Pr^z(z'|z)$ and $\Pr^\epsilon(\epsilon'|\epsilon)$ are characterized by stochastic matrices of dimensions $n_z \times n_z$ and $n_\epsilon \times n_\epsilon$ respectively. To keep parameterization of the processes parsimonious we use the method proposed in Rouwenhorst (1995) to discretize AR(1) processes for $\log(z)$ and $\log(\epsilon)$. This reduces the number of parameters to choose from $n_z(n_z-1)+n_\epsilon(n_\epsilon-1)$ to just 4, namely the standard deviation and persistence of each process: σ_z, ρ_z, σ_ϵ, ρ_ϵ. However, the persistence of the process (which determines the diagonal terms of the transition matrices) cannot be separately identified from the arrival rate of productivity or labor efficiency shocks (γ^z and γ^ϵ). We set the arrival rates to 1, and choose the persistence along with other parameters to match the set of targeted moments (discussed below). Finally, we set the mean of the processes to $\overline{z}$ and $\overline{\epsilon}$. We choose the values of $\overline{z}$ and $\overline{\epsilon}$ jointly with the other internally calibrated parameters discussed below.

Discretizing the processes in this way greatly simplifies the calibration of the model, although it comes at the cost of higher computational requirements since it makes the matrices involved in the finite difference method less sparse (Appendix C). In consequence we choose to set the grid size for z and ϵ to 11 and 13 nodes respectively. We experimented

	Externally Calibrated Parameters	
	Parameter	Value
$r^\star$	International Interest Rate	0
ρ	Discount Factor	0.0125
σ	CRRA Parameter	2
α	Technology - Capital Share	0.3
δ	Capital Depreciation	0
ν	Technology - Decreasing Returns	0.85
λ	Equity Constraint	1.42
γ^ϵ	Labor Efficiency - Arrival Rate	1
γ^z	Productivity - Arrival Rate	1

	Internally Calibrated Parameters	
	Parameter	Value
b	Unemployment Income	$w \cdot 10^{-5}$
γ^E	Job Destruction Arrival Rate	0.09
γ^U	Job Offer Arrival Rate - U	0.80
γ^S	Job Offer Arrival Rate - S	0.60
$\overline{\epsilon}$	Labor Efficiency- Base Value	0.10
σ_ϵ	Labor Efficiency - Variance	0.09
ρ_ϵ	Labor Efficiency - Persistence	0.90
$\overline{z}$	Productivity - Base Value	1.00
σ_z	Productivity - Variance	0.12
ρ_z	Productivity - Persistence	0.90

(a) Model Parameters

	Parameter	Value
$\underline{a}$	Borrowing Constraint	10^{-5}
$\overline{a}$	Asset Barrier	200
η_a	Asset Grid curvature	2
n_a	Asset Grid Size	120
n_ϵ	Labor Efficiency Grid Size	13
n_ϵ	Productivity Grid Size	11

(b) Computational Parameters

Table 5: Parameters

with finer grids and verified that our results do not depend on the particular size we chose. The other grid that affects the computation of the model is the grid on assets. We use a 120 node grid with curvature that increases density for low levels of assets. The limits of the grid are given by the borrowing constraint ($\underline{a}$) and an asset barrier ($\overline{a}$). We set $\underline{a} = 10^{-5}$, which ensures we don't face numerical issues, and we set $\overline{a} = 200$, as Figure 5b makes clear this upper bound does not affect the distribution of assets in the equilibrium of the model. The asset grid is given by:

$$a_i = \underline{a} + \left(\frac{i-1}{n_a - 1}\right)^{\eta_a} (\overline{a} - \underline{a}) \quad \text{for } i \in \{1, \ldots, n_a\}. \tag{17}$$

The values of all computational parameters are presented in Figure 5b.

Externally calibrated parameters We set a number of parameters to values common to the literature, or that match features of the Mexican economy independently of the equilibrium. Table 5a presents the parameter values we use in our baseline model.The discount factor is taken from Moll (2014) to match a 5 percent annual discount rate. To achieve a stationary distribution of assets we need $\rho - r^\star > 0$. We set this gap equal to 0.05 (at the annual frequency) as in Bianchi (2011), by setting $r^\star = 0$.[26]. The degree of

[26]Other studies have used lower values for the gap, for instance Itskhoki and Moll (2019) set it to 0.02.

decreasing returns (ν) is taken from Midrigan and Xu (2014). The curvature of the utility function (σ) is set to 2 and the power of capital in the self-employed production technology (α) is set to 1/3, consistent with standard values used in the literature. Finally, we set the equity constraint of the self-employed (λ) to match a debt to capital ratio of 42 percent, consistent with the Mexican data.

Internally calibrated parameters The remaining parameters are internally calibrated to jointly match a set of 11 moments from the Mexican data. The moments we target are the standard deviation of log-earnings for the self-employed and the employed, the composition of the workforce (share of unemployed, self-employed and employed), and the quarterly transition rates between occupations. Even though all parameters are calibrated jointly it is instructive to think of the variance of the stochastic processes (σ_z and σ_ϵ)as matching the standard deviation of earnings, the unemployment income (b) and the mean of the stochastic processes ($\bar{z}$ and $\bar{\epsilon}$) as matching the composition of the workforce, and the persistence of the stochastic processes (ρ_z and ρ_ϵ) and the arrival rates of shocks (γ^E, γ^U and γ^S) as matching the transition rates. The values of the parameters are presented in Table 5a, and the value of the targeted moments in the data and the model are presented in Table 6 which we discuss below.

Model performance Overall the model is able to match the targeted moments, as seen in Table 6. However, there are two important exceptions. The model overestimates the standard deviation of log-earnings for the self-employed, and it underestimates the transitions from unemployment to self-employment. The high variance of the earnings of the self-employed does not overly concern us, given that the data from the ENOE is likely to under-sample high earners, particularly those among the self-employed, biasing down the estimate of the standard deviation obtained from the data. Moreover, the higher variance for self-employment earnings plays two roles in the model in matching other relevant moments. First, it makes self-employment more attractive, helping to match the share of self-employed agents in the economy. Second, it increases the mass of "highly-productive" self-employed, which generates additional labor demand (this link between self-employment and labor demand is overly tight in the baseline model as explained in Section 3.6). On the other hand, we consider the low transition rate from unemployment to employment to be a shortcoming of the current parameterization of the model. We will discuss its implications in the next section as they become relevant.

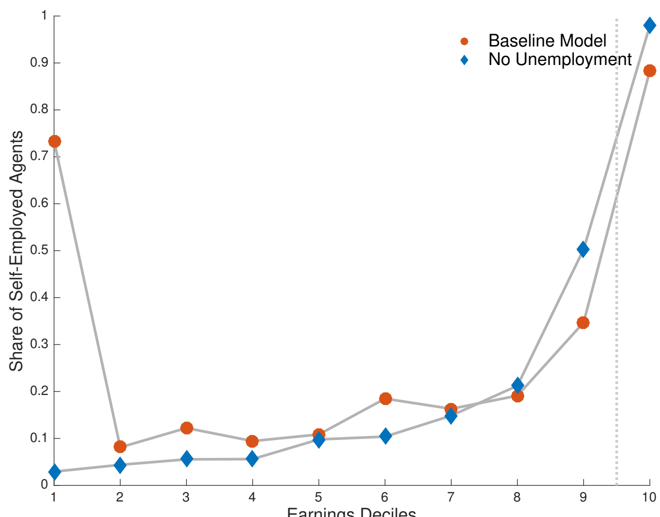
The model also fits relevant untargeted moments. Most importantly, the distribution of the self-employed by earnings. Figure 4 presents the share of self-employed for each decile of the earnings distribution. The model replicates the U-shaped relationship between earnings

Std.Dev. log-Earnings			Workforce (%)			Transitions (%)		
	Data	Model		Data	Model		Data	Model
S	0.44	0.88	U	2.6	2.2	U2S	26.9	6.8
E	0.25	0.26	S	29.5	28.5	U2E	47.1	52.3
			E	68.0	69.3	S2U	2.0	0.8
						S2E	19.2	14.5
						E2U	1.7	1.6
						E2S	8.1	6.1

Table 6: Moments for Calibration: Targets and Outcomes

Note: *The table reports all the moments targeted in the calibration. Targets are reported in the "Data" columns. The moments implied by the model are reported in the "Model" columns.*

Figure 4: Self-Employment rate by deciles of the earnings distribution



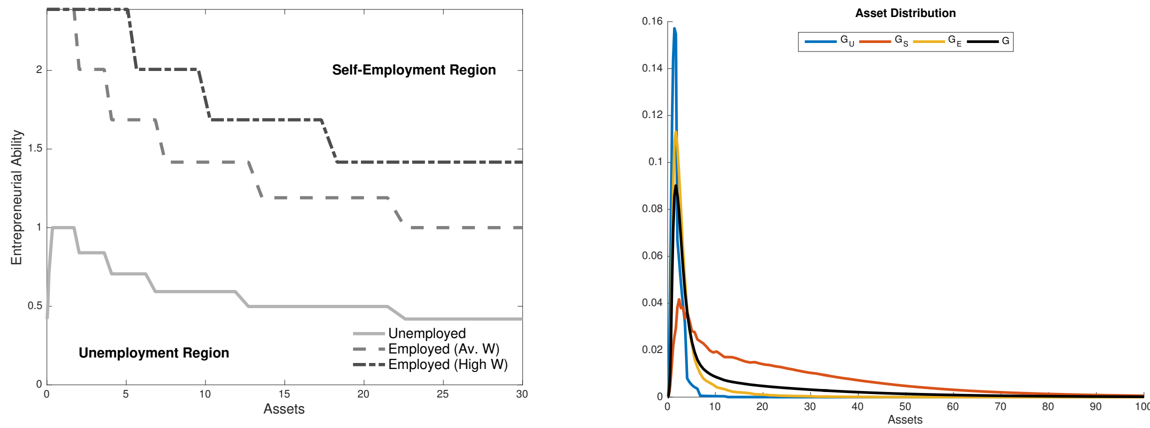
Note: *The figure reports the share of self-employed agents for each decile of the earnings distribution. The orange circles correspond to the baseline model with unemployment risk. The blue diamonds correspond to the alternative model without unemployment risk.*

and the self-employment rate, with the self-employed concentrated in the lower and upper ends of the earnings distribution. Yet, the share of self-employed agents for the top decile is overestimated relative to what was found in Figure 3. As we mentioned above, this is part due to the undersampling of high-earning self-employed individuals in the data, and in part due to the relatively high value of the variance of self-employed earnings in the model, which helps to generate more high-productivity self-employed.

The joint distribution of self-employment and earnings is an important feature of the data because of its power in distinguishing between different specifications of the model (in the spirit of Nakamura and Steinsson (2018)). While our baseline model with unemployment risk is consistent with the data, the simplified version of the model without unemployment risk, and free mobility between employment and unemployment is

Figure 5: Model Performance

(a) Self-Employment Occupational Choice (b) Wealth Distribution



Note: *The figures shows equilibrium outcomes of the baseline model. The left panel presents the occupational choice into self-employment. Lines depict the threshold value of productivity (z) for each level of assets (a) and a given value of labor efficiency (zero for the unemployed). The agent chooses self-employment if productivity is above the threshold. The right panel presents the wealth distribution of agents in the model along with the conditional wealth distributions by occupation.*

not. This model shares these features with most other models of self-employment in the macro-development literature (i.a. Buera et al., 2011, 2015, 2017; Midrigan and Xu, 2014). We calibrate the simplified model to match the same moments as our baseline model (save from those involving unemployment). Figure 4 makes clear that the model without unemployment is unable to capture the joint distribution of self-employment and earnings, specifically, the prevalence of low-earning self-employed agents. In this sense this model specification is rejected by the data. We return to this issue in Section 5.1 where we show that the simplified model is also at odds with the response of the economy to the implementation of job-guarantee programs.

The key to generate the mass of self-employed agents at the bottom of the earnings distribution is the occupational choice of the unemployed. Figure 5a presents the productivity thresholds for unemployed and employed agents in the model. The same mechanisms laid out in Section 1 translate into our baseline model (see Figure 1). Poor unemployed agents engage in self-employment regardless of their productivity. The model without unemployment risk cannot generate low-earning self-employed because the selection into self-employment ensures that only agents with high enough earnings opt into this occupation, just as the employed agents in Figure 5a. The success of the model in matching the patterns of self-employment comes from the behavior of the unemployed, coupled with a distribution of assets concentrated in the left tail (see Figure 5b).

5 Policy analysis

Self-employment is important for determining the response of the economy to various policies, in particular those intended to increase the productivity among enterprises of developing economies, or those providing insurance in the labor market. As hinted in Section 1, policies and self-employment interact through agent's occupational choices. Policies shape the incentives of agents to opt into self-employment, in turn changing the makeup of firms and workers in the economy. Moreover, the response of the economy to certain policies can prove helpful in distinguishing between different model specifications, and highlighting the relevant mechanisms at work. We expand on these themes in this section by examining the response of the model to three distinct policies: job guarantee programs, unemployment insurance and micro finance.

5.1 Job guarantee programs

Job guarantee programs are used across developing countries as a tool to provide income and potentially jump-start careers for individuals in high-unemployment regions. In general, these programs work by increasing labor demand through government provided jobs that pay (minimum) market wages and involve low-skilled tasks, typically in clerical or maintenance occupations in government outposts. Job guarantee programs like the NREGA program (described in Section 2 and Appendix A) can be massive. For instance, the NREGA program reached 53 million beneficiaries in 2010-2011 alone.

The scope of this type of government intervention raises questions about its implications for labor markets, in particular for the composition of the workforce and its effects on market wages. In particular, the effect on market wages changes the cost of providing programs of this nature, and is indicative of the level of slack in labor markets. In Appendix A we show that the implementation of the NREGA program is associated with a recomposition of the workforce away from self-employment and into more employment and more unemployment. Additionally, experimental evidence for India on the effects of labor demand shocks on wages shows that there is virtually no response of wages to changes in labor demand (Breza et al., 2017). The limited effect on wages of increases in labor demand point at the role of self-employment as a form of slack in the labor market. The intuition is that wages do not react to an increase in labor demand when agents are willing to opt out of self-employment and into salaried jobs at current wages.

Can the model reproduce the response in workforce composition and wages to an increase in labor demand? To test this we introduce government demand for labor n^{gov}, so that total labor demand is

$$N^d = \int n^\star(a, z) dG^S + n^{gov}. \tag{18}$$

We then solve for the new market clearing wage.[27] We show that the response of the economy to labor demand shocks, as in job guarantee programs, is informative about alternative specifications of the model. In particular, we show that unemployment risk, and the selection into self-employment it implies, play a crucial role in the model's ability to reproduce the economy's response to the labor demand shock.

In response to an increase in labor demand by the government the model implies a reallocation of the workforce from self-employment and into both higher employment and unemployment. Government demand amounts to 13 percent of the baseline demand for efficiency units of labor.[28] As in the findings of Imbert and Papp (2015), there is a strong crowding out effect over private employment (in this case over self-employment). Private demand for labor decreases 10.7 percent (self-employment decreases 1.7 percent). However, the overall demand for labor does increase (about 2.5 percent) as does the unemployment share of the workforce (0.1 percent). These movements are qualitatively in line with the findings of Breza et al. (2017). The decrease in self-employment comes mostly from the low-productive self-employed, because of this the productivity distribution improves relative to the baseline (in the first order stochastic dominance sense). Figure D.1 in Appendix D.2 presents more details.

Crucially, the increase in labor demand is not accompanied by an increase in wages. The market wage (per efficiency-unit of labor) increases 0.5 percent, implying an "Elasticity of wage to labor demand" ($^{\Delta\% w}/_{\Delta\% N^d}$) of 0.22. This is consistent with the experimental evidence referenced above that establishes a low response of wages to increases in labor demand. The presence of unemployment risk that induces low-earning self-employed plays a crucial role in generating this result. When we implement the increase in labor demand in the simplified model without unemployment risk (Appendix B) the increase in wages required to meet the additional demand is an order of magnitude larger than what is implied by the experimental evidence, implying a (percent) change in wages twice as large as the change in labor demand ($^{\Delta\% w}/_{\Delta\% N^d} = 2$). In this sense the response of the economy to job-guarantee programs is informative when determining the correct specification of the

[27] In the spirit of the experimental interventions cited above, and the implementation of the NREGA program, we do not introduce taxes to finance the government's labor demand.

[28] The size of the shock emulates the experiment by Breza et al. (2017).

model. As with the joint distribution of self-employment and earnings, the data rejects the model without unemployment risk in policy-relevant dimensions.

5.2 Unemployment insurance

Another policy often used by governments is the implementation of unemployment insurance or other forms of safety nets. Generally, these programs are intended to provide insurance to workers against unemployment spells. However, safety net programs affect the occupational choice of agents, in particular of the self-employed. The example in Section 1 highlights how access to unemployment insurance can deter unproductive agents from engaging in self-employment. While unemployment insurance is not typically thought of as an instrument to improve productivity, it can do so by changing the selection into entrepreneurship, and increasing labor supply.[29]

We model unemployment insurance as an increase in the income of unemployed agents, which is now $b+b_{UI}$.[30] The transfers to the unemployed are financed by (linear) labor taxes τ. We set b_{UI} to 5 percent of the minimum income among the employed. We set the value of τ to 0.15 percent, such that the government covers the expenses from unemployment insurance in equilibrium:

$$\tau \int w \cdot e^{\epsilon} dG^{E} = b_{UI} \cdot U. \tag{19}$$

Despite the policy providing relatively little income to the unemployed (5 percent of the minimum labor income), there are substantial changes in the composition of the workforce. As expected, unemployment increases (1.1 percentage points), but this change in unemployment also implies an increase in employment (as the unemployed receive job offers). Employment increases by 2.5 percentage points. These increases are matched by a decrease of 3.6 percentage points in the self-employed.

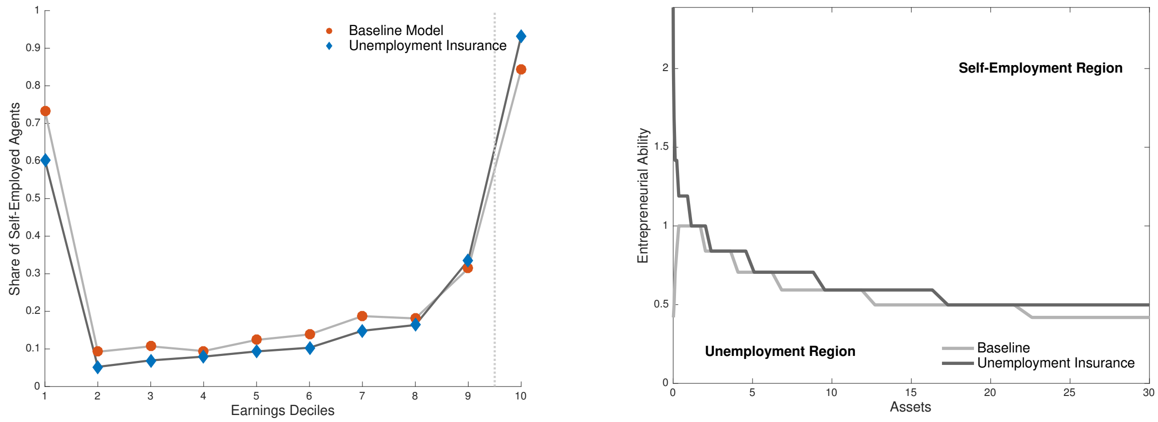
The reason behind the large reaction of the workforce is the effect of policy on the

[29]More generous safety nets can also improve productivity by spurring entrepreneurship. See evidence from France in Hombert et al. (2014, 2016), where the safety net provides insurance against productivity risk to new business owners, increasing firm creation. Unlike the case in this paper the additional insurance does not change selection into self-employment because the unemployed already had access to insurance.

[30]Our exercise excludes two features of unemployment insurance. First, there are no moral hazard problems in our model, as agent's search behavior is exogenous, although unemployment insurance does change the reservation wage of the unemployed. Second, we assume that the government can tell unemployed and self-employed agents apart when implementing the program. This need not be the case. If both unemployed and self-employed agents receive the payments, the program would not have the desired effect on occupational choice.

Figure 6: Model Performance - Unemployment Insurance

(a) Self-Employment by deciles of earnings　　(b) Self-Employment Occupational Choice



Note: *The figures show equilibrium outcomes of the model under unemployment insurance. The left panel reports the share of self-employed agents for each decile of the earnings distribution. The orange circles correspond to the baseline model. The blue diamonds correspond to the model with unemployment insurance. The right panel presents the occupational choice into self-employment of the unemployed. Lines depict the threshold value of productivity* (z) *for each level of assets* (a)*. The agent chooses self-employment if productivity is above the threshold.*

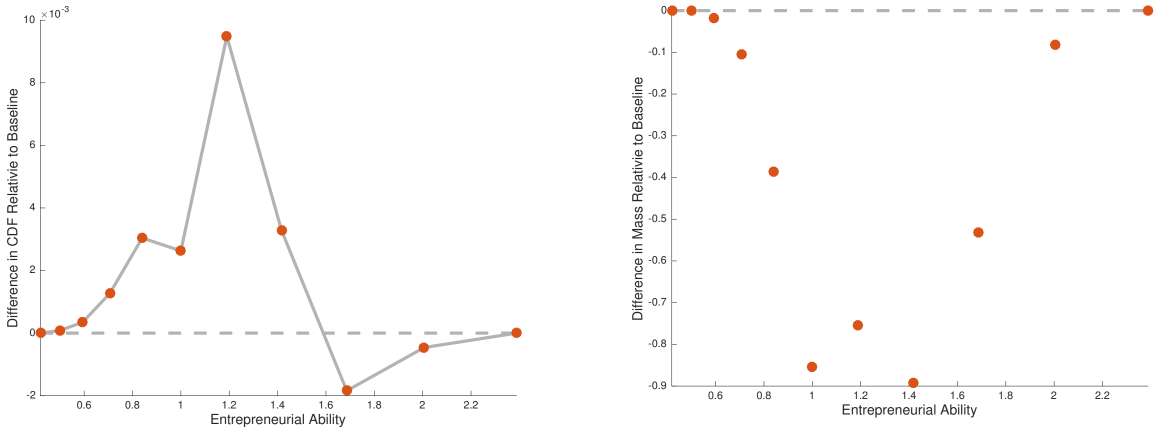
occupational choice of unemployed agents. See Figure 6b. As in the example of Section 1, the change in the occupational choice is concentrated among the poorest agents, who are now able to increase their minimum productivity threshold to become self-employed. Consequently, the reduction in self-employment is not uniform. Instead, it is concentrated among those at the bottom of earnings distribution, as observed in Figure 6a.

The change in selection into self-employment suggests an improvement in allocative efficiency in both labor and capital (having now more productive self-employed and more labor). This is in fact the case, and the increase in productivity is evidenced by an increase in output of 2.9 percent. The increase in output is considerable, taking into account the quantitatively small unemployment subsidy being given. As before, the reason behind the increase in output is the change in the selection into self-employment. Unemployment insurance reduces the mass of self-employed unevenly, concentrating the decreases among the lower half of the productivity distribution. This can be seen in Figure 7b. The consequence of this uneven change is an economy with relatively more of the more productive self-employed, as seen in the right end of Figure 7a that compares the CDF of productivity with and without insurance. More productive self-employed imply larger firms and more output.

The reason these low-productivity self-employed are better allocated in unemployment is the imperfection of self-employment as an insurance mechanism. The parameterization

Figure 7: Productivity Changes under Unemployment Insurance

(a) Cumulative Distributions (b) Mass per $z-$type



Note: *The figures show changes in the distribution of productivity* (z) *between the equilibrium of the baseline model and the unemployment insurance program. The left panel presents the difference in the CDF of productivity among the self-employed relative to the baseline model. Negative values indicate that the new distribution first order stochastically dominates the baseline distribution. The right panel presents the difference in the mass of self-employed agents for each* $z-$*type. Differences are due to changes in the distribution and overall mass of self-employed agents.*

of the model prefers lower job-offer arrival rates for those who are self-employed, compared to those who are unemployed. This means that, in the model, self-employment lowers the odds to find salaried employment. This is not a built-in assumption of the model, it is an outcome of the calibration. Also, this is a different statement than simply asserting that individuals transition at lower rates to employment from self-employment compared to unemployment, which could be the case even for equal job-arrival rates. Lower job-arrival rates from self-employment are consistent with cross-sectional findings by Jackson (2019), who report that engaging in gig-economy jobs in the U.S. causes individuals to find jobs at lower rates and have lower earnings in the long-run.

The results we obtain highlight how safety net programs can play a role in increasing productivity in developing countries by improving allocative efficiency. These programs not only allow individuals to search for longer and find better jobs, but they also prevent unproductive agents to engage in entrepreneurial activities for which they are ill-suited. This rationale works on top of the usual reasons for these type of programs as a source of insurance against labor income fluctuation.

5.3 Micro-finance

Finally, we use our baseline model to study the effects of micro-finance. Micro-finance is a common policy in developing economies, generally motivated by dysfunctional credit markets that prevent access to credit to entrepreneurs. These financial constraints curtail the growth of productive entrepreneurs. However, the experimental literature has found small average effects following the provision of micro-credit in developing countries (see among others, Banerjee, Duflo, Glennerster, and Kinnan (2015) and Banerjee, Breza, Duflo, and Kinnan (2019)). As we show below, the small average effects can be explained in part because micro-finance not only allows productive entrepreneurs to grow their business, but also makes self-employment more attractive for agents, regardless of productivity. As in the example of Section 1, this can reduce the productivity of the entrants into self-employment.

We model micro-finance as a loosening of the self-employed's collateral constraint. The micro-finance program gives an amount k_{mc} of seed capital to the poorest 10 percent of self-employed agents. For these agents the collateral constraint is:

$$k \leq \lambda \cdot a + k_{mc}. \tag{20}$$

We set k_{mc} to 5 percent of the average k among self-employed in the baseline. We assume that resources for seed capital come from abroad and that all loans have the same interest rate (the international rate $r^{\star}$).

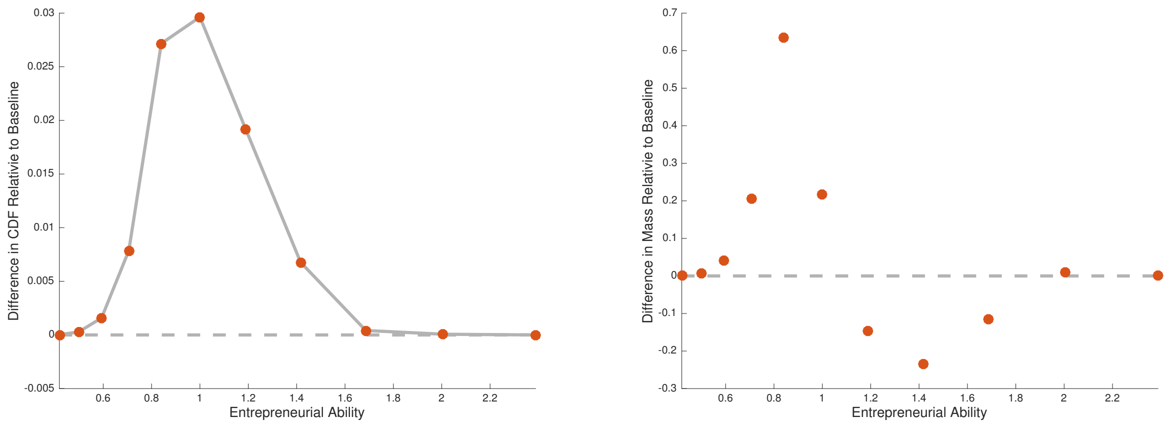
The magnitude of the effect of this policy over the composition of the workforce is small, mostly because it is a more targeted policy than the other two policies we studied. As expected, there is an increase in self-employment of 0.6 percentage points, which comes mostly from a decrease in unemployment of 0.4 percentage points. Employment also decreases (0.2 percentage points) making up the difference.

Because the policy is targeted at the self-employed with the lowest wealth (also the lowest earning) there is a (modest) increase in the share of self-employed in the lowest two deciles of the earnings distribution. There is also a differential effect over the occupational choice of the unemployed, reducing the productivity threshold to become self-employed, but only among the poorest. Figure D.2 in Appendix D.2 presents more details.

As a consequence of the implementation of the policy output increases 4.1 percent. But, is this increase due to more capital? or to higher productivity? Figure 8 describes what happens to the productivity distribution after the policy is enacted. Micro-finance makes self-employment more attractive for the poorest agents in the economy, and it decreases

Figure 8: Productivity Changes under Micro Finance

(a) Cumulative Distributions (b) Mass per $z-$type



Note: *The figures show changes in the distribution of productivity* (z) *between the equilibrium of the baseline model and the micro-finance program. The left panel presents the difference in the CDF of productivity among the self-employed relative to the baseline model. Negative values indicate that the new distribution first order stochastically dominates the baseline distribution. The right panel presents the difference in the mass of self-employed agents for each* $z-$*type. Differences are due to changes in the distribution and overall mass of self-employed agents.*

the productivity threshold to become self-employed. Because of this the entry into self-employment is concentrated among the low-productive, with even a small decrease in the mass of mid-productive self-employed (Figure 8b). The change in the selection into self-employment effectively worsens the productivity distribution with respect to the baseline (in the first order stochastic dominance sense). Figure 8a makes clear that following the implementation of the policy, output output increases despite a decrease in productivity.

We take the results of this model experiment as indicative of the importance of understanding selection into self-employment to determine the effects of policies like micro-finance. When self-employment acts as a refuge for individuals without access to other forms of income, policies that make engaging in entrepreneurial activities more attractive can have unintended consequences over the composition of the self-employed, potentially worsening the distribution of productivity.

6 Concluding remarks

Our main objective in doing this paper was to highlight the role of self-employment in developing economies in shaping the response of the economy to different policies (like job guarantee programs, unemployment insurance or micro-finance). In order to understand this response we show that is key to take into account how the policies change the selection of individuals into self-employment, in particular how unemployed agents react to the different policies. We see self-employment as critical to understand the reaction of the economy because it plays a role both for productive entrepreneurs starting businesses, and for poor agents that take on entrepreneurial activities as a last resource in their search for sources of income.

Moreover, self-employment is in itself a distinctive feature of developing economies. Self-employment is prevalent in developing economies, with rates much higher than those in richer countries. Additionally, self-employment is concentrated among low-earning individuals (as opposed to what happens in developed economies), and it is more likely to be taken up by those who are income-constrained (individuals who become self-employed out of necessity, rather than out of preference or aptitude). These features cannot be reproduced by the workhorse macro-development used in the literature to study the economy-wide impacts of development policies. We show that the introduction of unemployment risk, combined with financial frictions and lack of a social safety net, is key to reconcile the model with the features of self-employment in developing economies.

Unemployment risk shapes the selection of agents into self-employment, and in turn the response of the economy to various types of policies. By inducing a mass of low-earning self-employed, unemployment risk allows the model to reproduce the limited wage response of the economy to the implementation of job-guarantee programs. It also plays a crucial role in determining the effects of unemployment insurance and micro-finance on productivity. Here is where the role of selection becomes more relevant. Unemployment insurance prevents low-productivity individuals from engaging in entrepreneurial activities, increasing productivity as a result. On the contrary, micro-finance makes self-employment more attractive as a source of income for low-assets and low-productivity individuals, because of that productivity worsens, despite the positive effects of micro-finance over the productive self-employed.

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Appendices

A Evidence from India

We complement the analysis from Mexico that we presented in the previous sections with evidence from the National Rural Employment Guarantee Act (NREGA) program in India. The NREGA is a program that provides short-term jobs at market wages in rural India. The program was initially implemented in the poorer districts in India in 2006, and then extended in 2007 and 2008. The schedule in which the program was implemented across districts allows us to exploit regional variation to determine the effect on self-employment of a large workfare program. The exercise is similar to that of Imbert and Papp (2015), who study whether NREGA crowded out private sector employment. In our regressions we match data from the implementation of NREGA with microdata on time use and occupations from The National Sample Survey Office (NSSO).

Our exercise aims to test whether the creation of public salaried work positions (an increase in labor demand) reduces self-employment, while simultaneously increasing unemployment. We consider this type of response to be consistent with (a share of) individuals taking up self-employment out of necessity. In this case there would be no attachment to self-employment, and individuals would move out of self-employment and into the new salaried positions. Moreover, additional wage-earning individuals can allow households to spend more time searching for jobs (making unemployment tolerable). On the contrary, if self-employment was driven by preferences for or productivity in entrepreneurial activities, we would expect a decrease in unemployment with no changes in self-employment (also accompanied by an increase in wages in salaried work).

We estimate a dif-dif regression using the implementation of the NREGA across districts in India. Table A.1 summarizes the results of the exercise. The main finding is that in districts where the program was instituted first, the share of time dedicated to self-employment went down, and unemployment went up.[31] The results hold even when controlling for district and time fixed effects, and individual-level controls. Both the result on self-employment and on unemployment are consistent with self-employment being driven (partly) by individuals who are not particularly attached, or adept, to their entrepreneurial activities.

The exercise in this section differs with the previous in that it does not depend on individual variation, relying instead on variation at the regional level. Because of this reason we see this exercise as a good complement to the evidence of the previous sections. By comparing changes in self-employment across regions the results are not affected by

[31] We measure self-employment with the share of time dedicated to an individual's own business, instead of measuring it as a dichotomous state (as we did with the Mexican data). This change is motivated by the differences between life in rural India and urban Mexico. It is common for individuals in our Indian sample to be employed part of the week, and self-employed or unemployed for another part. Despite this differences, we actually see it as encouraging that the behavior of self-employment is consistent between the two countries.

	(1) SE	(2) SE	(3) SE	(4) U
NREGA	-0.012	-0.038***	-0.015*	0.010**
	[0.008]	[0.005]	[0.008]	[0.004]
Observations	395662	395662	395662	395662
Avg LHS	0.719	0.719	0.719	0.0516
District Fixed Effect	No	Yes	Yes	Yes
Individual Controls	Yes	Yes	Yes	Yes
Year-Quarter Fixed Effect	No	No	Yes	Yes
Constant	Yes	Yes	Yes	Yes

Table A.1: Change in self-empolyment due to implementation of NREGA

Note: *NREGA is the dif-dif coefficient that takes the value of one for all the district-year-quarter triplets in which the program is active. All the columns cluster the standard errors at the district level. Columns 1 to 3 analyze the effects on self-employment and differ on whether there are district and time fixed effects. Column four runs the same regression of column 3 but using unemployment as the dependent variable.*

differences in individual unobservable characteristics. The fact that this exercise (with a different design and data source) provides evidence consistent with the one from Mexico is encouraging. Nevertheless, we are cautious in interpreting our results. We only take this evidence as suggestive of the mechanism we highlighted in Section 1.[32]

Finally, the results in Table A.1 also relate to previous experimental results on labor rationing and slackness in local labor markets. In particular, our finding with the NREGA program are aligned with those of Breza et al. (2017), who randomize market-level transitory positive labor demand shocks across Indian villages to test the slackness of local labor markets. They find that, following their intervention, self-employment falls without an increase in the market wage. Their results are consistent with self-employed acting to hide the true slack of the market by harboring unemployed agents in need of income. We explore the effects of job-guarantee programs in our model, and also find a small reaction of wages coupled with reductions in self-employment in response to increases in labor demand (Section 5.1).

[32]There are various problems with our data that we cannot control for. For instance, misreporting of an individual's occupation, or presence of assortative mating. It is also hard to determine how comparable the public jobs offered by NREGA are to alternative positions. It is possible that our results are driven by NREGA jobs being considered superior.

B Model without unemployment risk

In this section we describe a simplified model without unemployment risk. The key difference from the model in Section 3 is that agents do not lose their labor efficiency when they opt into self-employment. As before an agent is free to become self-employed at any point, but now the agent is also free to become employed in response to changes in her labor efficiency, entrepreneurial ability or assets. There is no change in the stochastic process for labor efficiency, ϵ, and productivity, z, neither in the profit maximization problem of the self-employed, or in the definition of total supply and demand for labor.

B.1 Agent's Problem

Because agents can change instantly across occupations, the occupational choice problem reduces to maximizing instantaneous income. The value of an agent is then:

$$\rho V(a,z,\epsilon) = \max_{c} u(c) + V_a(a,z,\epsilon)\,\dot{a} + \gamma^{\epsilon}\int V(a,z,\epsilon')\,d\mathrm{Pr}^{\epsilon}(\epsilon'|\epsilon) + \gamma^{z}\int V(a,z',\epsilon)\,d\mathrm{Pr}^{z}(z'|z)$$
$$\text{s.t. } \dot{a} = \max\{we^{\epsilon}, \pi(a,z)\} + ra - c \qquad a \geq \underline{a} \tag{B.1}$$

with $\pi(a,z)$ given as in equation (8).

The optimal consumption decision is found as in Section 3:

$$c = u'^{-1}(V_a)\,. \tag{B.2}$$

Finally agents are classified as employed if $we^{\epsilon} \geq \pi(a,z)$ and as self-employed otherwise.

B.2 Equilibrium

An stationary equilibrium for this economy is a value function $\{V\}$, along with an optimal consumption function $\{c\}$, labor and capital demand from self-employed $\{n,k\}$, prices $\{r,w\}$ and a distribution of agents $\{G\}$ such that:

1. The value function satisfies (B.1).
2. Consumption (and thus asset accumulation) are consistent with the agent's optimization. That is, it is given by equation (B.2).
3. Capital and labor demand solve the self-employed's profit maximization problem. That is, they are given by (9) and (10) if $\nu < 1$, or by (11) if $\nu = 1$.
4. Labor market clears: $N^S = N^D$, where total labor supply is given by (12), and total labor demand by (13).
5. The interest rate is given by the international interest rate $r^{\star}$.
6. The distribution of agents is stationary. This is obtained if the distributions satisfy

the Kolmogorov Forward Equation (KFE):

$$\begin{aligned} 0 \; = & -\frac{\partial}{\partial a}\left[\dot{a} g\left(a, z, \epsilon\right)\right] - \gamma^{z} \int \Pr^{z}\left(z' | z\right) g\left(a, z, \epsilon\right) dz' + \gamma^{z} \int \Pr^{z}\left(z | z^{'}\right) g\left(a, z', \epsilon\right) dz' \\ & - \gamma^{\epsilon} \int \Pr^{\epsilon}\left(\epsilon' | \epsilon\right) g\left(a, z, \epsilon\right) d\epsilon' + \gamma^{\epsilon} \int \Pr^{\epsilon}\left(\epsilon | \epsilon^{'}\right) g\left(a, z, \epsilon'\right) d\epsilon' \end{aligned} \tag{B.3}$$

C Computational appendix

C.1 Solution to HJB equations

The model is solved using an implicit finite difference method as the one shown in Achdou et al. (2017). The occupational choice is solved through a splitting method, solving first for an auxiliary value $\tilde{V}$, the value that applies if the agent continues in the same occupation, and then solving for the occupational choice.

Consider grids over assets, entrepreneurial ability and labor efficiency:

$$\vec{a} = [a_1, \ldots, a_{n_a}] \qquad \vec{z} = [z_1, \ldots, z_{n_z}] \qquad \vec{\epsilon} = [\epsilon_1, \ldots, \epsilon_{n_\epsilon}]$$

with n_a, n_z and n_ϵ elements respectively, and constant distance between grid points of Δa, Δz and $\Delta \epsilon$. Let i denote the index of the asset dimension, j of the entrepreneurial ability, and k of the labor efficiency.

For notational convenience we will treat all value functions as depending on all three states, it is understood that V^U and V^S do not vary across ϵ. Denote $V^o_{ijk} = V^o(a_i, z_j, \epsilon_k)$ and let the backward and forward difference of the value function approximate the derivative:

$$V^o_a(a_i, z_j, \epsilon_k) \approx \frac{V^o_{i+1,jk} - V^o_{ijk}}{\Delta a} = \partial_a V^o_{ijk,F} \qquad V^o_a(a_i, z_j, \epsilon_k) \approx \frac{V^o_{ijk} - V^o_{i-1,jk}}{\Delta a} = \partial_a V^o_{ijk,B}\,.$$

The problem to solve is:

$$\rho \tilde{V}^E_{ijk} = u\left(c^E_{ijk}\right) + \partial_a V^E_{ijk} \cdot \left(y^E_{ijk} + r a_i - c^E_{ijk}\right) + \gamma^E \left(V^U_{ij} - \tilde{V}^E_{ijk}\right) + \gamma^z \sum_{j'=1}^{n_z} \left(V^E_{ij'k} - \tilde{V}^E_{ijk}\right) \Pr^z(z_{j'}|z_j) + \gamma^\epsilon \sum_{k'=1}^{n_\epsilon} \left(V^E_{ijk'} - \tilde{V}^E_{ijk}\right) \Pr^\epsilon(\epsilon_{k'}|\epsilon_k) \tag{C.1}$$

$$\rho \tilde{V}^U_{ij} = u\left(c^U_{ij}\right) + \partial_a V^U_{ij} \cdot \left(y^U_{ij} + r a_i - c^U_{ij}\right) + \gamma^U \sum_{k'=1}^{n_\epsilon} \left(V^E_{ijk'} - \tilde{V}^U_{ij}\right) \Pr^U(\epsilon_{k'})\, \mathbb{1}_{\left\{V^E_{ijk'} > V^U_{ij}\right\}} + \gamma^z \sum_{j'=1}^{n_z} \left(V^U_{ij'} - \tilde{V}^U_{ij}\right) \Pr^z(z_{j'}|z_j) \tag{C.2}$$

$$\rho \tilde{V}^S_{ij} = u\left(c^S_{ij}\right) + \partial_a V^S_{ij} \cdot \left(y^S_{ij} + r a_i - c^S_{ij}\right) + \gamma^S \sum_{k'=1}^{n_\epsilon} \left(V^E_{ijk'} - \tilde{V}^S_{ij}\right) \Pr^S(\epsilon_{k'})\, \mathbb{1}_{\left\{V^E_{ijk'} > V^S_{ij}\right\}} + \gamma^z \sum_{j'=1}^{n_z} \left(V^S_{ij'} - \tilde{V}^S_{ij}\right) \Pr^z(z_{j'}|z_j) \tag{C.3}$$

together with:

$$V^E_{ijk} = \max\left\{\tilde{V}^E_{ij,k}, \tilde{V}^U_{ij}, \tilde{V}^S_{ij}\right\} \tag{C.4}$$

$$V^U_{ij} = \max\left\{\tilde{V}^U_{ij}, \tilde{V}^S_{ij}\right\} \tag{C.5}$$

$$V^S_{ij} = \max\left\{\tilde{V}^U_{ij}, \tilde{V}^S_{ij}\right\} \tag{C.6}$$

The implicit method solves the following equation on $\tilde{V}^{o,n+1}_{ijk}$ given a value for $V^{o,n}_{ijk}$. For employment the equation is:

$$\begin{aligned}\frac{\tilde{V}^{E,n+1}_{ijk} - \tilde{V}^{E,n}_{ijk}}{\Delta} + \rho\tilde{V}^{E,n+1}_{ijk} = u\left(c^{E,n}_{ijk}\right) + \partial_a\tilde{V}^{E,n+1}_{ijk}\cdot s^{E,n}_{ijk} + \gamma^E\left(\tilde{V}^{U,n+1}_{ij} - \tilde{V}^{E,n+1}_{ijk}\right) \\ + \gamma^z\sum_{j'=1}^{n_z}\left(\tilde{V}^{E,n+1}_{ij'k} - \tilde{V}^{E,n+1}_{ijk}\right)\mathrm{Pr}^z\left(z_{j'}|z_j\right) \\ + \gamma^\epsilon\sum_{k'=1}^{n_\epsilon}\left(\tilde{V}^{E,n+1}_{ijk'} - \tilde{V}^{E,n+1}_{ijk}\right)\mathrm{Pr}^\epsilon\left(\epsilon_{k'}|\epsilon_k\right)\end{aligned} \tag{C.7}$$

For $o \in \{U, S\}$ the equation is:

$$\begin{aligned}\frac{\tilde{V}^{o,n+1}_{ij} - \tilde{V}^{o,n}_{ij}}{\Delta} + \rho\tilde{V}^{o,n+1}_{ij} = u\left(c^{o,n}_{ij}\right) + \partial_a\tilde{V}^{o,n+1}_{ij}\cdot s^{o,n}_{ij} + \sum_{k'=1}^{n_\epsilon}\tilde{\gamma}^{o,n}_{ijk'}\left(\tilde{V}^{E,n+1}_{ijk'} - \tilde{V}^{o,n+1}_{ij}\right)\mathrm{Pr}^o\left(\epsilon_{k'}\right) \\ + \gamma^z\sum_{j'=1}^{n_z}\left(\tilde{V}^{o,n+1}_{ij'} - \tilde{V}^{o,n+1}_{ij}\right)\mathrm{Pr}^z\left(z_{j'}|z_j\right)\end{aligned} \tag{C.8}$$

Note that the (known) value at iteration n is used to compute consumption, and the drift of the assets, which we will call savings for convenience:

$$s^{o,n}_{ijk} = y^o_{ijk} + ra_i - c^{o,n}_{ijk} \qquad \text{where} \qquad c^{o,n}_{ijk} = u'^{-1}\left(\partial_a V^{o,n}_{ijk}\right)$$

It is also used to define if the agent is willing to change after a job offer. We have:

$$\tilde{\gamma}^{U,n}_{ijk} = \gamma^U \mathbb{1}_{\left\{V^{E,n}_{ijk'} > V^{U,n}_{ij}\right\}} \qquad \tilde{\gamma}^{S,n}_{ijk} = \gamma^S \mathbb{1}_{\left\{V^{E,n}_{ijk'} > V^{S,n}_{ij}\right\}}$$

Next it is necessary to determine whether to use the forward or backward approximation to the first derivatives of the value function. We follow the "upwind scheme" presented in Achdou et al. (2017).

Since consumption can be defined with the backward or forward difference approximation we get:

$$s^{o,n}_{ijk,B} = y^o_{ijk} + ra_i - u'^{-1}\left(\partial_a V^{o,n}_{ijk,B}\right) \qquad s^{o,n}_{ijk,F} = y^o_{ijk} + ra_i - u'^{-1}\left(\partial_a V^{o,n}_{ijk,F}\right)$$

The idea is to use the backward difference when the implied drift is negative, and the forward difference when the drift is positive. Yet there are cases for which $s^{o,n}_{ijk,F} < 0 < s^{o,n}_{ijk,B}$, in these cases we set savings equal to zero, so the derivative is not used, in any case the FOC of the problem gives the exact derivate of the value function as: $\partial_a \overline{V}^{o,n}_{ijk} = u'(y_{jk} + ra_i)$.[33]

Consumption is then:

$$c^{o,n}_{ijk} = u'^{-1}\left(\partial_a V^{o,n}_{ijk,B}\right)\mathbb{1}_{\left\{s^{o,n}_{ijk,B}<0\right\}} + u'^{-1}\left(\partial_a V^{o,n}_{ijk,F}\right)\mathbb{1}_{\left\{s^{o,n}_{ijk,F}>0\right\}} + \left(y^o_{ijk} + ra_i\right)\mathbb{1}_{\left\{s^{o,n}_{ijk,F}<0<s^{o,n}_{ijk,B}\right\}},$$

and the drift term for assets is replaced by:

$$\begin{aligned}
\partial_a \tilde{V}^{o,n+1}_{ijk} \cdot s^{o,n}_{ijk} &= \partial_a \tilde{V}^{o,n+1}_{ijk,B}\left[s^{o,n}_{ijk,B}\right]^- + \partial_a \tilde{V}^{o,n+1}_{ijk,F}\left[s^{o,n}_{ijk,F}\right]^+ \\
&= \frac{\tilde{V}^{o,n+1}_{ijk} - \tilde{V}^{o,n+1}_{i-1,jk}}{\Delta a}\left[s^{o,n}_{ijk,B}\right]^- + \frac{\tilde{V}^{o,n+1}_{i+1,jk} - \tilde{V}^{o,n+1}_{ijk}}{\Delta a}\left[s^{o,n}_{ijk,F}\right]^+
\end{aligned}$$

Grouping terms we get the following expression for employment:

$$\begin{aligned}
\frac{\tilde{V}^{E,n+1}_{ijk} - \tilde{V}^{E,n}_{ijk}}{\Delta} + \rho \tilde{V}^{E,n+1}_{ijk} = u\left(c^{E,n}_{ijk}\right) &+ \gamma^E \tilde{V}^{U,n+1}_{ij} \\
&+ x^E_{ijk}\tilde{V}^{E,n+1}_{ijk} + x^{E-}_{ijk}\tilde{V}^{E,n+1}_{i-1,jk} + x^{E+}_{ijk}\tilde{V}^{E,n+1}_{i+1,jk} \\
&+ \gamma^z \sum_{j'=1}^{n_z} \Pr^z\left(z_{j'}|z_j\right)\tilde{V}^{E,n+1}_{ij'k} + \gamma^\epsilon \sum_{k'=1}^{n_\epsilon} \Pr^\epsilon\left(\epsilon_{k'}|\epsilon_k\right)\tilde{V}^{E,n+1}_{ijk'}
\end{aligned}$$

where

$$\begin{aligned}
x^E_{ijk} &= \frac{\left[s^{E,n}_{ijk,B}\right]^-}{\Delta a} - \frac{\left[s^{E,n}_{ijk,F}\right]^+}{\Delta a} - \gamma^E - \gamma^z - \gamma^\epsilon \\
x^{E-}_{ijk} &= -\frac{\left[s^{E,n}_{ijk,B}\right]^-}{\Delta a} \\
x^{E+}_{ijk} &= \frac{\left[s^{E,n}_{ijk,F}\right]^+}{\Delta a}
\end{aligned}$$

[33] Additional care is needed because of the non-convexities introduced by the occupational choice of agents. It is possible that both $s^{o,n}_{ijk,B} < 0$ and that $s^{o,n}_{ijk,F} > 0$ for the same state. In this case we take the drift that provides the highest change in value by comparing $u\left(c^{o,n}_{ijk,B}\right) + \partial_a V^{o,n}_{ijk,B} \cdot s^{o,n}_{ijk,B}$ with $u\left(c^{o,n}_{ijk,F}\right) + \partial_a V^{o,n}_{ijk,F} \cdot s^{o,n}_{ijk,F}$. We omit this from the notation for readability.

For unemployment and self-employment:

$$\frac{\tilde{V}_{ij}^{o,n+1}-\tilde{V}_{ij}^{o,n}}{\Delta}+\rho\tilde{V}_{ij}^{o,n+1}=u\left(c_{ij}^{o,n}\right)+\sum_{k'=1}^{n_\epsilon}\tilde{\gamma}_{ijk'}^{o,n}\Pr^{o}\left(\epsilon_{k'}\right)\tilde{V}_{ijk'}^{E,n+1}$$
$$+x_{ij}^{o}\tilde{V}_{ij}^{o,n+1}+x_{ij}^{o-}\tilde{V}_{i-1,j}^{o,n+1}+x_{ij}^{o+}\tilde{V}_{i+1,j}^{o,n+1}$$
$$+\gamma^{z}\sum_{j'=1}^{n_z}\Pr^{z}\left(z_{j'}|z_j\right)\tilde{V}_{ij'}^{o,n+1}$$

where

$$x_{ij}^{o}=\frac{\left[s_{ij,B}^{o,n}\right]^{-}}{\Delta a}-\frac{\left[s_{ij,F}^{o,n}\right]^{+}}{\Delta a}-\sum_{k'=1}^{n_\epsilon}\tilde{\gamma}_{ijk'}^{o,n}\Pr^{o}\left(\epsilon_{k'}\right)-\gamma^{z}$$
$$x_{ij}^{o-}=-\frac{\left[s_{ij,B}^{o,n}\right]^{-}}{\Delta a}$$
$$x_{ij}^{o+}=\frac{\left[s_{ij,F}^{o,n}\right]^{+}}{\Delta a}$$

C.1.1 Boundary Conditions

A final loose end before writing up the linear system in matrix form is what to do with the boundaries of the different grids. At the lower boundary of the asset grid the agent is subject to a no-borrowing constraint. Hence it has to be the case that the agent does not try to borrow. The drift has to be non-negative at that point, which implies that $\tilde{x}_{1jk}^{o-}=0$ for all (j,k). At the upper boundary a similar constraint can be imposed, so that $\tilde{x}_{n_a jk}^{o+}=0$. This should arise naturally if the upper boundary is high enough. Notice that imposing these restrictions implies that V_{0j}^{n+1} and $V_{n_a+1,j}^{n+1}$ are not part of the system.

C.1.2 System Solution

The equations above describe a system of $n_a\times n_z\left(2+n_\epsilon\right)$ equations, its best to define the value function a stack of three value functions, one for each occupation:

$$V=\left[V^{U};V^{S};V^{E}\right]^{T}$$
$$V^{o}=\text{vec}\left[V_{ijk}^{o}\right]$$

The system is:

$$\frac{1}{\Delta}\left(V^{n+1}-V^{n}\right)+\rho V^{n+1}=u^{n}+A^{n}V^{n+1}$$

where $u^{n}=\left[u^{U,n};u^{S,n};u^{E,n}\right]$ and $u^{o,n}=\text{vec}\left[u\left(c_{ijk}^{o,n}\right)\right]$ with consumption computed as explained above.

Matrix A^n is given by:

$$A^n = B^n + C + D$$

$$B^n = \begin{bmatrix} B^n_{UU} & \mathbf{0} & B^n_{UE} \\ \mathbf{0} & B^n_{SS} & B^n_{SE} \\ B_{EU} & \mathbf{0} & B^n_{EE} \end{bmatrix} \qquad C = \begin{bmatrix} \tilde{C} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{C}_E \end{bmatrix} \qquad D = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{D} \end{bmatrix}$$

The matrices B^n_{oo} are sparse and they only contain elements in the diagonal, upper diagonal and lower diagonal. Consider $X_o = \left[\tilde{x}^o_{ijk}\right]$, $X^-_o = \left[\tilde{x}^{o,-}_{ijk}\right]$ and $X^+_o = \left[\tilde{x}^{o,+}_{ijk}\right]$, all three dimensional matrix that contain the coefficients $\tilde{x}$ (note that $\tilde{x}$ is already adjusted for the boundaries). Then we have: $\operatorname{diag}(B^n_{oo}) = \operatorname{vec}(X_o)$, $\operatorname{diag}^+(B^n_{oo}) = \operatorname{vec}(X^+_o)$ and $\operatorname{diag}^-(B^n_{oo}) = \operatorname{vec}(X^-_o)$, where the upper diagonal and lower diagonal are adjusted not to include the zero terms of the boundaries.

The matrices $B^n_{oo'}$ depend on the type of transition. For the transition from employment to unemployment we have:

$$B_{EU} = \gamma^E \begin{bmatrix} I_{n_a \cdot n_z} \\ \vdots \\ I_{n_a \cdot n_z} \end{bmatrix}$$

so that B_{EU} is of size $n_a \cdot n_z \cdot n_\epsilon \times n_a \cdot n_z$. For the transition from unemployment and self-employment to employment:

$$B^n_{oE} = \gamma^o \left[\ \Pr^o(\epsilon_1) \operatorname{diag}\left(\operatorname{vec}\left(\mathbb{1}_{\left\{V^E_{ij1} > V^o_{ij}\right\}}\right)\right) \quad \cdots \quad \Pr^o(\epsilon_{n_\epsilon}) \operatorname{diag}\left(\operatorname{vec}\left(\mathbb{1}_{\left\{V^E_{ijn_\epsilon} > V^o_{ij}\right\}}\right)\right)\ \right]$$

where we abuse notation by letting $\operatorname{diag}(\cdot)$ give a diagonal matrix when it is evaluated in a vector.

Matrices $\tilde{C}$ and $\tilde{D}$ are also sparse and they are independent of the iteration. Their construction takes advantage of the fact that the elements of $\tilde{C}$ only vary with j and the elements of $\tilde{D}$ only vary with k. We first construct $\tilde{C} = \gamma^z \Pr^z \otimes I_{n_a}$ and $\tilde{C}_E = \gamma_z I_{n_\epsilon} \otimes \tilde{C}$. Finally, $\tilde{D} = \gamma^\epsilon \Pr^\epsilon \otimes I_{n_a \cdot n_z}$.

This problem can now be expressed as:

$$T^n V^{n+1} = t^n$$

where:

$$T^n = \left(\frac{1}{\Delta} + \rho\right) I_{n_a n_z (2+n_\epsilon)} - A^n \qquad t^n = u^n + \frac{1}{\Delta} V^n$$

C.1.3 Algorithm

1. Compute matrices C and D. These matrices do not change with equilibrium prices or iterations.
2. Take as given w.
3. Solve for earnings in each state: y^o_{ijk} for each combination of (a, z, ϵ) and occupation. These values don't change with iterations.

4. Guess a value for V^n, a $n_a n_z (2 + n_\epsilon)$ vector. It is easier to store it as three separate matrices of dimensions $n_a \times n_z$, $n_a \times n_z$ and $n_a \times n_\epsilon \times n_z$.
   (a) We find it better to find the initial condition by solving for a fixed point of the problem without occupational choice (this same algorithm without the last step).
5. Compute the backward and forward drift: $s_{ijk,B}^{o,n}$ and $s_{ijk,F}^{o,n}$ for $i = \{2, \dots, n_a\}$ and $i = \{1, \dots, n_a - 1\}$ respectively, and all (j, k, o).
$$s_{ijk,B}^{o,n} = y_{jk}^o + ra_i - u'^{-1}\left(\partial_a V_{ijk,B}^{o,n}\right) \qquad s_{ijk,F}^{o,n} = y_{jk}^o + ra_i - u'^{-1}\left(\partial_a V_{ijk,F}^{o,n}\right)$$
   These values are stored in six matrices (two per occupation, one with backward drift and the other one with forward drift).
6. For all (i, j, k, o) compute consumption as:
$$c_{ijk}^{o,n} = u'^{-1}\left(\partial_a V_{ijk,B}^{o,n}\right) 1_{s_{ijk,B}^{o,n}<0} + u'^{-1}\left(\partial_a V_{ijk,F}^{o,n}\right) 1_{s_{ijk,F}^{o,n}>0} + \left(y_{jk}^o + ra_i\right) 1_{s_{ijk,F}^{o,n}<0<s_{ijk,B}^{o,n}}$$
   These values are stored in three matrices of dimensions $n_a \times n_z$, $n_a \times n_z$ and $n_a \times n_\epsilon \times n_z$.
7. Compute the utility vector as: $u^n = \left[u^{U,n}; u^{S,n}; u^{E,n}\right]$ and $u^{o,n} = \text{vec}\left[u\left(c_{ijk}^{o,n}\right)\right]$.
8. Compute the adjusted shock arrival rates:
$$\tilde{\gamma}_{ijk}^{U,n} = \gamma^U 1_{V_{ijk}^{E,n} > V_{ijk}^{U,n}} \qquad \tilde{\gamma}_{ijk}^{S,n} = \gamma^S 1_{V_{ijk}^{E,n} > V_{ijk}^{U,n}} \qquad \tilde{\gamma}_{ijk}^{E,n} = \gamma^E$$
9. Compute the matrices $X_o = \left[\tilde{x}_{ijk}^o\right]$, $X_o^- = \left[\tilde{x}_{ijk}^{o,-}\right]$ and $X_o^+ = \left[\tilde{x}_{ijk}^{o,+}\right]$.
10. Compute matrix $B^n = \begin{bmatrix} B_{UU}^n & \mathbf{0} & B_{UE}^n \\ \mathbf{0} & B_{SS}^n & B_{SE}^n \\ B_{EU}^n & \mathbf{0} & B_{EE}^n \end{bmatrix}$, where $\text{diag}\left(B_{oo}^n\right) = \text{vec}\left(X_o\right)$, $\text{diag}^+\left(B_{oo}^n\right) = \text{vec}\left(X_o^+\right)$ and $\text{diag}^-\left(B_{oo}^n\right) = \text{vec}\left(X_o^-\right)$, where the upper diagonal and lower diagonal are adjusted not to include the zero terms of the boundaries. The matrices $B_{oo'}^n$ are defined above.
11. Compute the matrix $A^n = B^n + C + D$.
12. Compute the matrix T and vector t:
$$T^n = \left(\frac{1}{\Delta} + \rho\right) I_{3n_a n_\epsilon n_z} - A^n \qquad t^n = u^n + \frac{1}{\Delta} V^n$$
13. Compute $V^{n+1/2}$ as:
$$V^{n+1/2} = \left(T^n\right)^{-1} t^n$$
   (a) We use the Biconjugate gradients stabilized (l) method, preconditioned with LU Factorization. See Matlab functions "ilu" and "bicgstabl."
14. Divide the vector $V^{n+1/2}$ into three matrices of $n_a \times n_z$, $n_a \times n_z$ and $n_a \times n_\epsilon \times n_z$: $V^{U,n+1/2}$, $V^{S,n+1/2}$, and $V^{E,n+1/2}$.

15. Compute $V^{U,n+1}$, $V^{S,n+1}$, and $V^{E,n+1}$ as follows:

$$V_{ijk}^{U,n+1} = \max\left\{V_{ijk}^{U,n+1/2}, \tilde{V}_{ijk}^{S}\right\}$$
$$V_{ijk}^{S,n+1} = \max\left\{V_{ijk}^{U,n+1/2}, V_{ijk}^{S,n+1/2}\right\}$$
$$V_{ijk}^{E,n+1} = \max\left\{V_{ijk}^{U,n+1/2}, \tilde{V}_{ijk}^{S}, V_{ijk}^{E,n+1/2}\right\}$$

(a) Define the following matrices as indicators of the occupation choice: $\left[\tilde{\chi}_{ijk}^{oo'}\right]$

$$\tilde{\chi}_{ij}^{US} = \begin{cases} 1 & \text{if } V_{ij}^{U,n+1} = \tilde{V}_{ij}^{S} \\ 0 & \text{otw} \end{cases} \qquad \tilde{\chi}_{ij}^{SU} = \begin{cases} 1 & \text{if } V_{ij}^{S,n+1} = V_{ij}^{U,n+1/2} \\ 0 & \text{otw} \end{cases}$$

$$\tilde{\chi}_{ijk}^{EU} = \begin{cases} 1 & \text{if } V_{ijk}^{E,n+1} = V_{ij}^{U,n+1/2} \\ 0 & \text{otw} \end{cases} \qquad \tilde{\chi}_{ijk}^{ES} = \begin{cases} 1 & \text{if } V_{ijk}^{E,n+1} = \tilde{V}_{ij}^{S} \\ 0 & \text{otw} \end{cases}$$

These functions are 1 if the agent changes occupations at (i, j, k).

(b) Define now the vectors $\chi^{oo'} = \text{vec}\left(\tilde{\chi}^{oo'}\right)$ to be used later. χ is a vector of length $n_a n_z (2 + n_\epsilon)$.

C.2 Solution to KFE equations

Before solving the KFE the transition matrix A has to be modified to include the endogenous transitions between unemployment and self-employment. For this we use the indicators χ constructed as part of the value function iteration.

Now, consider a transition matrix P:

$$P = \begin{bmatrix} P^{UU} & P^{US} & A^{UE} \\ P^{SU} & P^{SS} & A^{SE} \\ P^{EU} & P^{ES} & A^{EE} \end{bmatrix}$$

note that since there are not endogenous transitions to employment the last column of matrices are just as in matrix A. The other columns are modified only if there are endogenous transitions. Note that each matrix $P^{oo'}$ is of size $n_a n_\epsilon n_z \times n_a n_\epsilon n_z$.

1. Make all matrices $P^{oo'} = A^{oo'}$ and $P^{oo} = A^{oo}$.
2. For matrix P make zero any (column) entry related to an endogenous transition, since these states are not reached. For all m and q in $\{1, \ldots, n_a n_\epsilon n_z\}$:

$$P_{mq}^{*U} = 0 \qquad \text{if } \chi^{US}(q) = 1$$

$$P_{mq}^{*S} = 0 \qquad \text{if } \chi^{SU}(q) = 1$$

$$P_{mq}^{*E} = 0 \qquad \text{if } \chi^{EU}(q) = 1 \quad \text{or} \quad \chi^{ES}(q) = 1$$

where $* \in \{U, S, E\}$.

3. For matrix P adjust entries to take into account endogenous transitions coming from other occupation o into occupation $o^{'}$. This implies moving the columns of P^{o*} that were set to 0 because of transitions into $P^{*o^{'}}$. For all m and q in $\{1, \ldots, n_a n_\epsilon n_z\}$:

$$P^{*S}_{m,q-l_q} = P^{*S}_{m,q-l_q} + A^{*U}_{mq} \qquad \text{if } \chi^{US}(q) = 1$$

$$P^{*U}_{mq} = P^{*U}_{mq} + A^{*S}_{mq} \qquad \text{if } \chi^{SU}(q) = 1$$
$$P^{*U}_{mq} = P^{*U}_{mq} + A^{*E}_{mq} \qquad \text{if } \chi^{EU}(q) = 1$$
$$P^{*S}_{m,q-l_q} = P^{*S}_{m,q-l_q} + A^{*E}_{mq} \qquad \text{if } \chi^{ES}(q) = 1$$

where l_q maps the index of the agent after paying the l_k units of adjustment cost.

4. Finally as explained in Moll's example for stopping time (multiple assets with adjustment costs) the diagonal elements with transitions have to be adjusted:

$$P^{UU}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{US}(m) = 1$$

$$P^{SS}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{SU}(m) = 1$$

$$P^{EE}_{mm} = \frac{-1}{\Delta} \qquad \text{if } \chi^{EU}(m) = 1 \quad \text{or} \quad \chi^{ES}(m) = 1$$

Moll says: "To see why the $^{-1}/_{\Delta}$ term shows up, consider the time-discretized process for g:

$$\dot{g}_t = P^T g_t \longrightarrow g_{t+\Delta t} = (\Delta P + I)^T g_t$$

where I is the identity matrix. The transition matrix $\tilde{P} = \Delta P + I$ needs to have all entries in the adjustment region $\tilde{C}_{mm} = 0$ and hence $\Delta P + I = 0$. Without the adjustment, the matrix P is singular.

The system to solve is:

$$P^T g = 0$$

A simple way to solve the system is to make one of the elements of g to be equal to an arbitrary number, and replace such row of P^T by a row of zeros with a one in the diagonal. Call this matrix $\tilde{P}^T$ and let $\tilde{\iota} = [0, \ldots, 0, 0.1, 0, \ldots, 0]^T$ then solve for:

$$\tilde{g} = \left[\tilde{P}^T\right]^{-1} \tilde{\iota}$$

Normalize $\tilde{g}$ so that it sums to 1: $\tilde{g} = \tilde{g}/_{\text{sum}(\tilde{g})}$. Finally define g as:

$$g_i = \frac{\tilde{g}_i}{\Delta a_i}$$

D Additional graphs and tables

D.1 Mobility across occupations

In what follows we dig deeper into how the labor market status of an individual affects transitions. To do so, we follow the same strategy as Katz and Krueger (2017) in their study of alternative work arrangements in the U.S.. The first question we ask is whether unemployment makes an individual more likely to transition into self-employed. To answer this we focus on the universe of individuals who are either employed or self-employed in period t, and check whether the transitions to self-employment are larger for those agents who were unemployed in the previous period.

This exercise differs from the conditional transition rates reported in Table 2. The regression we conduct allows us to control for the (observable) characteristics of individuals, thus comparing transition rates across similar individuals (in terms of age, education and location), instead of computing transitions among individuals with the same labor market status (e.g. unemployed). Of course, our results do not control for all characteristics of the individuals (in particular, we do not observe wealth in our data), neither can we control for unobservable traits that make some individuals more employable, or more inclined to start their own business. We thus take this evidence as only suggestive of the mechanisms we study.

Table D.1 reports the regression results. The transition rates of unemployed agents to self-employment are 20.9 percentage points higher than those exhibited by (observationally) comparable agents who had a salaried job. This result holds after controlling for age, education, and after adding time and city fixed-effects.[34] While we are not able to control for all the relevant factors affecting transitions into self-employment, we interpret the higher transition rate from unemployment as suggestive of the role of self-employment as an outside option for individuals who need an income source, as opposed to self-employment representing entrepreneurial activities for which the individual is better suited (relative to working in a salaried job). In Section 2.2 and Appendix A we provide more evidence consistent with this interpretation.

The second question we ask is whether opting into self-employment hurts an individual's chances to find a job. The effect of self-employment on the transitions into employment matters to determine how persistent the effects of individual occupational choices are, and how policies that affect those choices affect in turn the labor market. To answer this question we focus on the universe of agents who are either unemployed or self-employed in period $t-1$, and follow them to determine whether or not they become employed. As before this allows us to compare the transition rates of self-employed individuals with (observationally) comparable unemployed individuals.

Table D.2 presents the regression results. We do in fact find that self-employed

[34] Our results align with those of Katz and Krueger (2017) for the U.S.. They find that unemployed individuals are more likely to transition to an alternative work arrangement job than agents who are employed. Alternative work arrangements (e.g. working for Uber or Task Rabbit) have a similar role as self-employment in Mexico, namely offering a self-procured source of income.

	(1) SE	(2) SE	(3) SE	(4) SE
U_{t-1}	0.209***	0.209***	0.208***	0.208***
	[0.003]	[0.003]	[0.003]	[0.003]
S_{t-1}	0.717***	0.717***	0.706***	0.706***
	[0.012]	[0.012]	[0.012]	[0.012]
Age			0.002***	0.002***
			[0.000]	[0.000]
Constant	0.080***	0.109***	-0.027	-0.038
	[0.005]	[0.005]	[395.990]	[167.520]
Observations	1033397	1033397	1033397	1033397
Mean Ent	0.285	0.285	0.285	0.285
Schooling Controls	No	No	Yes	Yes
City Fixed Effect	No	No	No	Yes
Time Fixed Effect	No	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes

Table D.1: Transitions to Self-Employment

Note: *The LHS variable is an indicator variable that takes the value of one if the individual is self-employed and zero if the individual is employed. U_{t-1} and S_{t-1} are indicator variables that take the value of 1 if the individual was unemployed or self employed in the previous quarter respectively. Age is the age in years. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. The sample consists of individuals who are either employed or self-employed in period t. We run the regressions by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

individuals are 34 percentage points less likely to transition to employment than comparable unemployed individuals. Even if the actual effect of self-employment is not as large, this estimate indicates that opting into self-employment can have long-lasting implications for an individual, particularly for low-productivity self-employed who are now less likely to abandon this state and move to employment. We will revisit this when analysing the quantitative fit of the model in Section 4.

An important caveat for the results in Table D.2 is that we are not controlling for selection into self-employment on the basis of entrepreneurial ability or preference for self-employment. We also lack the full occupational history of individuals, so we cannot condition on the attachment to self-employment of each individual. However, we can partially address some of these shortcomings by focusing on individuals who were unemployed to begin with (in period $t-2$), and who then either remain unemployed or transition to self-employment (in period $t-1$). This lets us compare individuals that start in a common state, and who then differ on whether or not they transition through self-employment. When we perform this analysis the same result emerges, with individuals who become self-employed being 14.4 percentage points less likely to transition to employment in period t. The regression results can be found in Table D.3.

Finally, Tables D.4 and D.5 present results for regressions of different (self-reported) search activities of the unemployed.

	(1)	(2)	(3)	(4)	(5)
	E	E	E	E	E
S_{t-1}	-0.268***	-0.268***	-0.255***	-0.254***	-0.340***
	[0.021]	[0.021]	[0.020]	[0.020]	[0.014]
Age			-0.006***	-0.006***	-0.004***
			[0.000]	[0.000]	[0.000]
Second Earner					0.022
					[0.018]
$S_{t-1}\times$ Second Earner					0.024**
					[0.011]
Constant	0.463***	0.417***	0.704***	0.684***	0.589
	[0.015]	[0.014]	[0.103]	[0.111]	[1203.540]
Observations	327250	327250	327250	327250	145945
Mean Emp	0.221	0.221	0.221	0.221	0.221
Schooling Controls	No	No	Yes	Yes	Yes
Time Fixed Effect	No	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

Table D.2: Transitions to Employment

Note: *The LHS variable is an indicator variable that takes the value of one if the individual is employed in period t. S_{t-1} and Second Earner are indicator variables that take the value of 1 if the individual was self-employed and if the individual's couple was an income earner in the previous quarter respectively. Age is the age in years. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. The sample consists of individuals who were either unemployed or self-employed in period $t-1$. We run the regressions by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

	(1)	(2)	(3)	(4)	(5)
	E	E	E	E	E
S_{t-1}	-0.066***	-0.069***	-0.084***	-0.086***	-0.144***
	[0.014]	[0.014]	[0.017]	[0.017]	[0.050]
Age			-0.009***	-0.009***	-0.007***
			[0.001]	[0.001]	[0.001]
Second Earner					0.035
					[0.057]
$S_{t-1}\times$ Second Earner					-0.024
					[0.058]
Constant	0.383***	0.456***	0.854***	0.863***	1.327***
	[0.017]	[0.064]	[0.226]	[0.228]	[0.075]
Observations	7320	7320	7320	7320	3205
Mean Emp	0.355	0.355	0.355	0.355	0.355
Schooling Controls	No	No	Yes	Yes	Yes
Time Fixed Effect	No	Yes	Yes	Yes	Yes
Weighted	Yes	Yes	Yes	Yes	Yes

Table D.3: Transitions to Employment from Unemployment

Note: *The LHS variable is an indicator variable that takes the value of one if the individual is employed in period t. S_{t-1} and Second Earner are indicator variables that take the value of 1 if the individual was self-employed and if the individual's couple was an income earner in the previous quarter respectively. Age is the age in years. Standard errors are clustered at the city level. Schooling controls are a set of dummies by education level to control non-parametrically for education. Time fixed effects are at the year-quarter level. The sample consists of individuals who were unemployed in period $t-2$, and were not employed in period $t-1$. We run the regressions by weighted OLS. *, **, and ***, denote significance at the 10%, 5%, and 1% level.*

	(1) Asked	(2) Job Post	(3) Public Ag	(4) Temp	(5) SE Plans	(6) Internet	(7) Newspaper	(8) Need to Work	(9) Age
Second Earner	-0.033	-0.009	-0.006	-0.002	0.002	-0.058***	-0.022	0.000	1.564***
	[0.023]	[0.010]	[0.008]	[0.003]	[0.001]	[0.019]	[0.017]	[0.000]	[0.528]
Constant	0.203***	0.021**	0.020**	0.008***	0.001	0.105***	0.069***	0.000	41.063***
	[0.022]	[0.009]	[0.008]	[0.002]	[0.001]	[0.018]	[0.016]	[.]	[0.497]
Observations	11214	11214	11214	11214	11214	11214	11214	11214	11214

Table D.4: Second Earner and Job-Search Activities

Note: *The LHS variable is an indicator variable that takes the value of one if individual i performed the given activity to search for a job in the previous quarter. The last two columns correspond to weather or not the individual declares to have a need to work, and differences in age. Second Earner is an indicator variable that takes the value of one if the individual's couple was an income earner in period* $t-1$*. Standard errors are clustered at the city level. All regressions include schooling controls (a set of dummies by education level to control non-parametrically for education), and time fixed effects at the year-quarter level. The sample consists of individuals who were unemployed in period* $t-1$*. The regressions are run by weighted OLS.* *, **, *and* ***, *denote significance at the 10%, 5%, and 1% level.*

	(1) Asked	(2) Job Posting	(3) Public Ag.	(4) Temp	(5) SE Plans	(6) Internet	(7) Newspaper	(8) Need to Work	(9) Age
Remittances	0.183*	0.023	0.004	-0.007***	-0.004**	-0.035***	0.092	0.045	-0.250
	[0.103]	[0.023]	[0.012]	[0.002]	[0.002]	[0.004]	[0.089]	[0.051]	[2.385]
Constant	0.157***	0.007***	0.008***	0.007***	0.004**	0.035***	0.040***	0.043***	43.766***
	[0.007]	[0.002]	[0.002]	[0.002]	[0.002]	[0.004]	[0.004]	[0.004]	[0.212]
Observations	8200	8200	8200	8200	8200	8200	8200	8200	8200

Table D.5: Remittances and Job Search Activities

Note: *The LHS variable is an indicator variable that takes the value of one if individual i performed the given activity to search for a job in the previous quarter. The last two columns correspond to weather or not the individual declares to have a need to work, and differences in age. Remittances is an indicator variable that takes the value of one if the individual reported having received remittances in period* $t-1$. *Standard errors are clustered at the city level. All regressions include schooling controls (a set of dummies by education level to control non-parametrically for education), and time fixed effects at the year-quarter level. The sample consists of individuals who were unemployed in period* $t-1$. *The regressions are run by weighted OLS.* *, **, *and* ***, *denote significance at the 10%, 5%, and 1% level.*

D.2 Model: additional graphs and tables

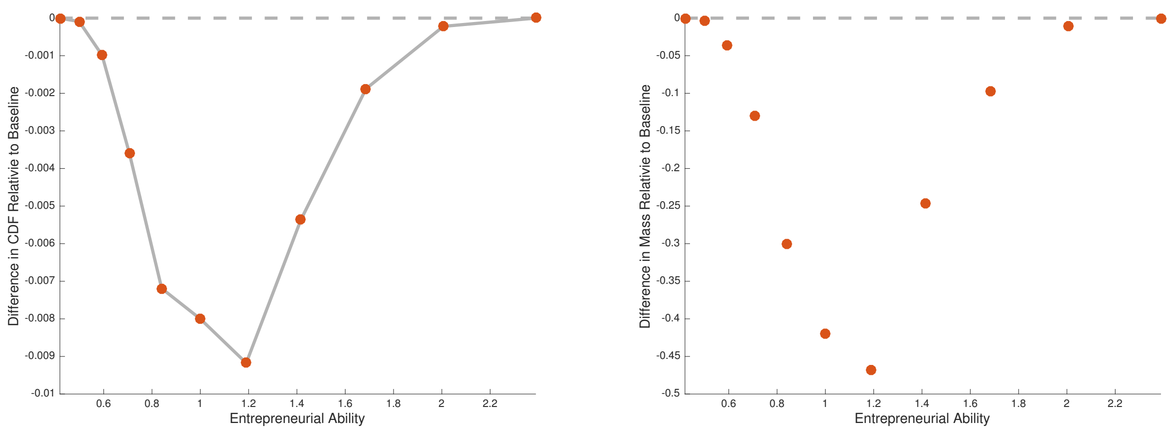
Shocks		Preferences	
$\overline{\epsilon}$	Labor Efficiency - Base Value	ρ	Discount Factor
σ_ϵ	Labor Efficiency - Variance	σ	CRRA Parameter
ρ^ϵ	Labor Efficiency - Persistence	Technology	
γ^ϵ	Labor Efficiency - Arrival Rate	α	Capital Share
$\overline{z}$	Productivity - Base Value	δ	Capital Depreciation
σ_z	Productivity - Variance	ν	Decreasing Returns
ρ^z	Productivity - Persistence	Assets	
γ^z	Productivity - Arrival Rate	λ	Equity Constraint
γ^U	Job Offer Arrival Rate - Unemployed	$\underline{a}$	Borrowing Constraint
γ^S	Job Offer Arrival Rate - Self-employed	$\overline{a}$	Asset Barrier
γ^E	Job Destruction Arrival Rate	Income	
		b	Unemployed Income

Table D.6: Model Parameters

Figure D.1: Productivity Changes under Job Guarantee

(a) Cumulative Distributions

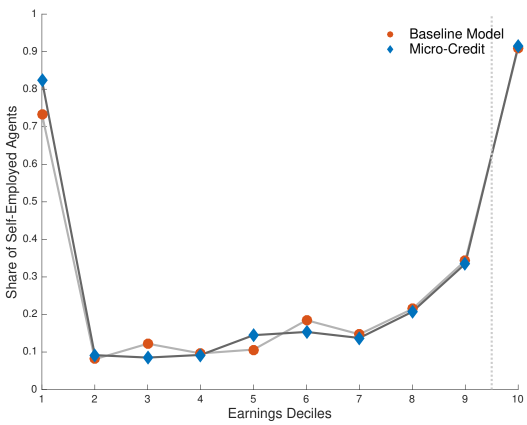
(b) Mass per $z-$type



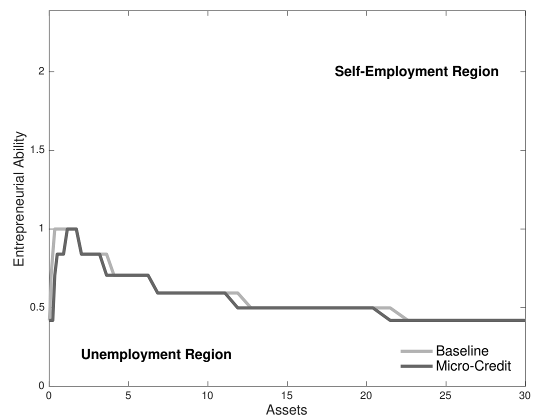
Note: *The figures shows changes in the distribution of productivity (z) between the equilibrium of the baseline model and the job guarantee program. The left panel presents the difference in the CDF of productivity among the self-employed relative to the baseline model. Negative values indicate that the new distribution first order stochastically dominates the baseline distribution. The right panel presents the difference in the mass of self-employed agents for each $z-$type. Differences are due to changes in the distribution and overall mass of self-employed agents.*

Figure D.2: Model Performance - Micro-Finance

(a) Self-Employment by deciles of earnings



(b) Self-Employment Occupational Choice



Note: *The figures show equilibrium outcomes of the model under micro-finance. The left panel reports the share of self-employed agents for each decile of the earnings distribution. The orange circles correspond to the baseline model. The blue diamonds correspond to the model with micro-finance. The right panel presents the occupational choice into self-employment of the unemployed. Lines depict the threshold value of productivity* (z) *for each level of assets* (a)*. The agent chooses self-employment if productivity is above the threshold.*