Beyond Ranks
Inequality in the Measurement of Mobility

Rory McGee Sergio Ocampo

University of Western Ontario

July, 2026

Mobility and Inequality are Intertwined

- Many mobility measures focus on individuals' *relative positions*:
  - Changes in income ranks (Bartholomew 1973; Shorrocks 1978)
  - Rank correlations (Schiller 1977; Chetty, Hendren, Kline, Saez, and Turner 2014; Fagereng, Mogstad, and Rønning 2021; many others!)

Mobility and Inequality are Intertwined

- Many mobility measures focus on individuals' *relative positions*:
  - Changes in income ranks (Bartholomew 1973; Shorrocks 1978)
  - Rank correlations (Schiller 1977; Chetty, Hendren, Kline, Saez, and Turner 2014; Fagereng, Mogstad, and Rønning 2021; many others!)
- Purely ordinal measures discard *cardinal information*.
  - Same rank movement can represent very different material changes
  - Invariant to all monotone transformations of income (including progressive taxation)
- Purely cardinal measures miss the role of *relative position* (Galton 1886; Hart 1983; Solon 1992; Zimmerman 1992; Hart–Shorrocks 1993; Fields–Ok 1996; Ray–Genicot 2023; Davis–Mazumder 2024; many others!)

Mobility and Inequality are Intertwined

- Many mobility measures focus on individuals' *relative positions*:
  - Changes in income ranks (Bartholomew 1973; Shorrocks 1978)
  - Rank correlations (Schiller 1977; Chetty, Hendren, Kline, Saez, and Turner 2014; Fagereng, Mogstad, and Rønning 2021; many others!)
- Purely ordinal measures discard *cardinal information*.
  - Same rank movement can represent very different material changes
  - Invariant to all monotone transformations of income (including progressive taxation)
- Purely cardinal measures miss the role of *relative position* (Galton 1886; Hart 1983; Solon 1992; Zimmerman 1992; Hart–Shorrocks 1993; Fields–Ok 1996; Ray–Genicot 2023; Davis–Mazumder 2024; many others!)

Our objective: Measure mobility using both position and material income differences

Measuring Inequality

The Lorenz curve encodes the relative income distribution (Atkinson 1970; Sen 1973)

- Economy with income vector $Y = (y_1, \ldots, y_N)$ and mean income μ,

$$r(y, Y) \equiv \frac{1}{N} \sum_{j=1}^{N} \mathbb{1}\{y_j \leq y\}, \qquad L(r, Y) \equiv \sum_{\{j : r_j \leq r\}} \frac{y_j}{N\mu}.$$

- $L_i = L(r_i, Y)$ is the cumulative income share held by dynasties no richer than i
  - Lorenz ordinates are scale invariant (Shorrocks 1993) and bounded in [0,1]

Measuring Inequality

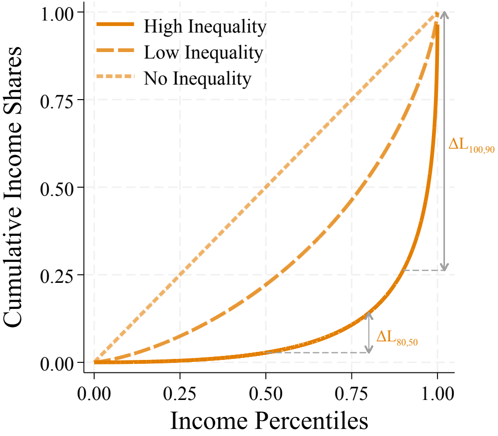
The Lorenz curve encodes the relative income distribution (Atkinson 1970; Sen 1973)

- Economy with income vector $Y = (y_1, \ldots, y_N)$ and mean income μ,

$$r(y, Y) \equiv \frac{1}{N} \sum_{j=1}^{N} \mathbb{1}\{y_j \leq y\}, \qquad L(r, Y) \equiv \sum_{\{j: r_j \leq r\}} \frac{y_j}{N\mu}.$$

- $L_i = L(r_i, Y)$ is the cumulative income share held by dynasties no richer than i
    - Lorenz ordinates are scale invariant (Shorrocks 1993) and bounded in [0,1]
- Lorenz ordinates combine *position* (r_i) with the *relative distribution of income*
    - A measure of "relative abundance" (Runciman, 1966; Kakwani 1984) or "aspirational status"

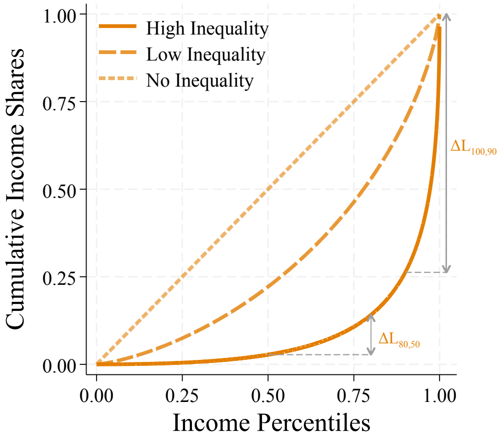
Positional Mobility in Unequal Economies



In unequal economies, income distributions compress at the bottom and spread at the top.

Lorenz ordinates capture this relationship:

Positional Mobility in Unequal Economies



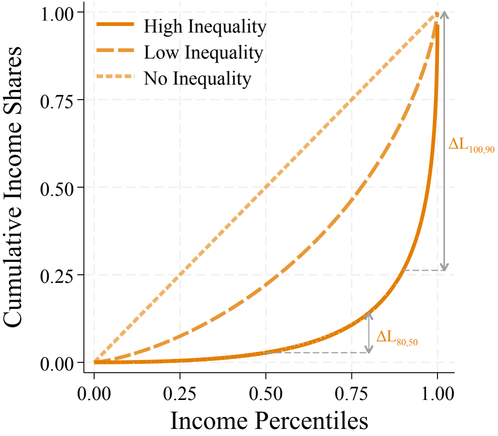
In unequal economies, income distributions compress at the bottom and spread at the top.

Lorenz ordinates capture this relationship:

- Differences in Lorenz ordinates shrink when income differences are small

→ Large rank changes + Small Lorenz change

Positional Mobility in Unequal Economies



In unequal economies, income distributions compress at the bottom and spread at the top.

Lorenz ordinates capture this relationship:

- Differences in Lorenz ordinates shrink when income differences are small

→ Large rank changes + Small Lorenz change

- Differences in Lorenz ordinates spread when income differences are large

→ Small rank changes + Large Lorenz change

Lorenz Mobility: $m_i = L_i^K - L_i^P$

Change in how much of economy's income lies beneath—or above—you

$\longrightarrow$ Income distance from the bottom and to the top

Lorenz Mobility: $m_i = L_i^K - L_i^P$

Change in how much of economy's income lies beneath—or above—you

$\longrightarrow$ Income distance from the bottom and to the top

- Lorenz mobility combines cardinality of own income and inequality of distribution
  1. When income goes up Lorenz ordinate goes up (holding constant income distribution)
  2. Evaluate economy-wide changes in income in the light of changes in inequality

Lorenz Mobility: $m_i = L_i^K - L_i^P$

Change in how much of economy's income lies beneath—or above—you

$\longrightarrow$ Income distance from the bottom and to the top

- Lorenz mobility combines cardinality of own income and inequality of distribution
  1. When income goes up Lorenz ordinate goes up (holding constant income distribution)
  2. Evaluate economy-wide changes in income in the light of changes in inequality
- Ranks can miss both cardinal and distributional changes
  - No rank changes when comparing *co-monotone* income distributions
  - Equivalent rank mobility in market vs disposable income under progressive taxation

Inequality, Redistribution, and Mobility

Inequality, Redistribution, and Mobility

Lorenz mobility explicitly links inequality and mobility

Inequality, Redistribution, and Mobility

Lorenz mobility explicitly links inequality and mobility

- Look at the effect of Pigou-Dalton transfers
  - Redistribute income to reduce inequality
  - Take dynasties h and j with $y_h < y_j$ and transfer $\delta > 0$ from j to h with $y_h + \delta \leq y_j - \delta$
  - Respect order between h and j but can generate other positional changes

Inequality, Redistribution, and Mobility

Lorenz mobility explicitly links inequality and mobility

- Look at the effect of Pigou-Dalton transfers
    - Redistribute income to reduce inequality
    - Take dynasties h and j with $y_h < y_j$ and transfer $\delta > 0$ from j to h with $y_h + \delta \leq y_j - \delta$
    - Respect order between h and j but can generate other positional changes
- Contrast with income swaps
    - Change dynasties' positions without changing inequality

Inequality, Redistribution, and Mobility

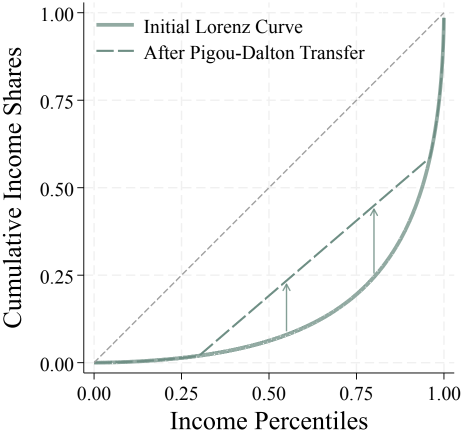
Lorenz mobility explicitly links inequality and mobility

- Look at the effect of Pigou-Dalton transfers
  - Redistribute income to reduce inequality
  - Take dynasties h and j with $y_h < y_j$ and transfer $\delta > 0$ from j to h with $y_h + \delta \leq y_j - \delta$
  - Respect order between h and j but can generate other positional changes
- Contrast with income swaps
  - Change dynasties' positions without changing inequality

Movements towards equality increase upward mobility

Redistribution: Pigou–Dalton Transfers

▸ Proportional transfers



Reducing inequality raises the Lorenz curve

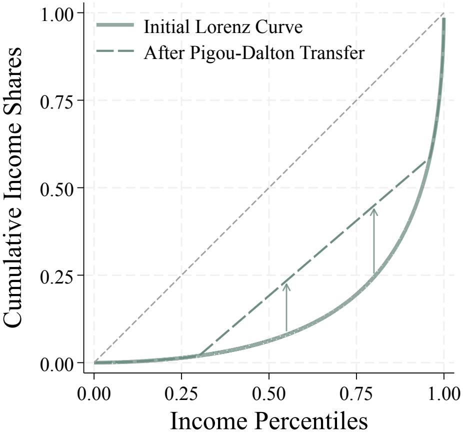
(Atkinson 1970)

- Upward mobility if rank is preserved
- Transfers can also change positions!

Net result depends on positional mobility …

Redistribution: Pigou–Dalton Transfers

▸ Proportional transfers



Reducing inequality raises the Lorenz curve

(Atkinson 1970)

- Upward mobility if rank is preserved
- Transfers can also change positions!

Net result depends on positional mobility …

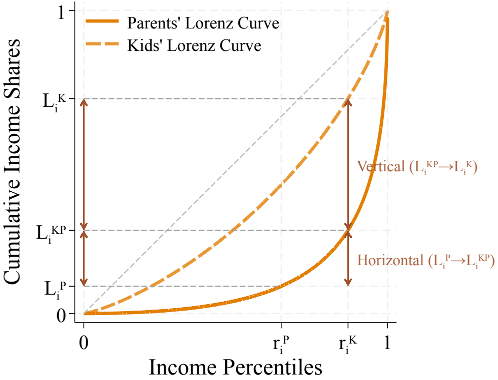
Mobility necessarily higher for all dynasties relative to equivalent income swaps

- Upward movers move farther up
- Downward movers fall by less

Disentangling Changes in Inequality and Changes in Position

Vertical and Horizontal Mobility

$$m_i = L_i^K - L_i^P = \underbrace{L_i^K - L_i^{KP}}_{\text{Vertical}} + \underbrace{L_i^{KP} - L_i^P}_{\text{Horizontal}}.$$



Horizontal Mobility:

Change in rank holding inequality constant

- Movement along the parents' Lorenz curve

Vertical Mobility:

Change in inequality holding position fixed

- Movement between Lorenz curves

Co-monotone generations have only vertical mobility;
Identical distributions have only horizontal mobility.

Aggregating Mobility

Aggregating Mobility (Kolmogorov-Nagumo-de Finetti theorem)

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} m_i^{\gamma} \right)^{1/\gamma}$$

Aggregating Mobility (Kolmogorov-Nagumo-de Finetti theorem)

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} m_i^{\gamma} \right)^{1/\gamma}$$

Dynastic mobility:

details

Signed mobility: $m_i = L_i^K - L_i^P$,

Symmetric mobility: $m_i = \left| L_i^K - L_i^P \right|$

Aggregating Mobility (Kolmogorov-Nagumo-de Finetti theorem)

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} m_i^{\gamma} \right)^{1/\gamma} = \underbrace{\overline{m}}_{\text{Average Mobility}} \times \overbrace{\mathcal{C}_{\gamma}(M)}^{\text{Mobility Concentration}}$$

Dynastic mobility:

details

Signed mobility: $m_i = L_i^K - L_i^P$, Symmetric mobility: $m_i = \left| L_i^K - L_i^P \right|$

Average Mobility and Mobility Concentration:

details

- Tradeoff between levels and inequality of mobility (SWF: Kolm 1969; Atkinson 1970; Sen 1973)

Aggregating Mobility (Kolmogorov-Nagumo-de Finetti theorem)

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} m_i^{\gamma} \right)^{1/\gamma} = \underbrace{\overline{m}}_{\text{Average Mobility}} \times \overbrace{\mathcal{C}_{\gamma}(M)}^{\text{Mobility Concentration}}$$

Dynastic mobility:

▸ details

Signed mobility: $m_i = L_i^K - L_i^P$,

Symmetric mobility: $m_i = \left| L_i^K - L_i^P \right|$

Average Mobility and Mobility Concentration:

▸ details

- Tradeoff between levels and inequality of mobility (SWF: Kolm 1969; Atkinson 1970; Sen 1973)
- Curvature γ controls role of concentration:

$\gamma < 1$: penalizes concentrated mobility

$\gamma > 1$: favors large, concentrated movements

Average Mobility and Inequality: Signed Mobility

$$\text{Net Mobility:} \quad \overline{m} \equiv \frac{1}{N}\sum_{i=1}^{N}\left(L_i^K - L_i^P\right) = \frac{1}{2}\underbrace{\left(\mathcal{G}(Y^P) - \mathcal{G}(Y^K)\right)}_{\text{Change in Gini Coefficient}}$$

- Decreases in inequality are increases in (net) mobility
- Completes the mobility–inequality link (even when the Lorenz curves of Y^K and Y^P intersect)
- Net mobility is *vertical mobility*; horizontal mobility is zero-sum

Average Mobility and Inequality: Signed Mobility

Net Mobility: $$\overline{m} \equiv \frac{1}{N}\sum_{i=1}^{N}\left(L_i^K - L_i^P\right) = \frac{1}{2}\underbrace{\left(\mathcal{G}(Y^P) - \mathcal{G}(Y^K)\right)}_{\text{Change in Gini Coefficient}}$$

- Decreases in inequality are increases in (net) mobility
- Completes the mobility–inequality link (even when the Lorenz curves of Y^K and Y^P intersect)
- Net mobility is *vertical mobility*; horizontal mobility is zero-sum

Panel independence: Cross-sectional data is sufficient for mobility (Ray & Genicot, 2023)

Average Mobility and Inequality: Signed Mobility

Net Mobility: $$\overline{m} \equiv \frac{1}{N}\sum_{i=1}^{N}\left(L_i^K - L_i^P\right) = \frac{1}{2}\underbrace{\left(\mathcal{G}(Y^P) - \mathcal{G}(Y^K)\right)}_{\text{Change in Gini Coefficient}}$$

- Decreases in inequality are increases in (net) mobility
- Completes the mobility–inequality link (even when the Lorenz curves of Y^K and Y^P intersect)
- Net mobility is *vertical mobility*; horizontal mobility is zero-sum

Panel independence: Cross-sectional data is sufficient for mobility (Ray & Genicot, 2023)

Symmetric mobility: Additionally captures upward/downward mobility ▸ details

Analytical Example: Pareto Income and HSV Taxes

$$y^G \sim \text{Pareto}(\alpha^{\boldsymbol{G}}) \quad \longrightarrow \quad \text{Gini Coefficient: } \mathcal{G}(Y^G) = \frac{1}{2\alpha^{\boldsymbol{G}} - 1}$$

Analytical Example: Pareto Income and HSV Taxes

$$y^G \sim \text{Pareto}(\boldsymbol{\alpha}^{\boldsymbol{G}}) \quad \longrightarrow \quad \text{Gini Coefficient: } \mathcal{G}(Y^G) = \frac{1}{2\boldsymbol{\alpha}^{\boldsymbol{G}} - 1}$$

- Taxes are progressive so that disposable income is log-linear in market income

(Feldstein 1969; Benabou 2002; Heathcote, Storesletten, Violante 2014)

$$\widehat{y}_i^G = \lambda^G (y_i^G)^{1-\tau^G} \quad \longrightarrow \quad \widehat{\boldsymbol{\alpha}}^{\boldsymbol{G}} = \frac{\alpha^G}{1-\tau^G}$$

Analytical Example: Pareto Income and HSV Taxes

$$y^G \sim \text{Pareto}(\alpha^G) \longrightarrow \text{Gini Coefficient: } \mathcal{G}(Y^G) = \frac{1}{2\alpha^G - 1}$$

- Taxes are progressive so that disposable income is log-linear in market income

(Feldstein 1969; Benabou 2002; Heathcote, Storesletten, Violante 2014)

$$\widehat{y}_i^G = \lambda^G (y_i^G)^{1-\tau^G} \longrightarrow \widehat{\alpha}^G = \frac{\alpha^G}{1-\tau^G}$$

Net mobility: $\widehat{\widehat{m}} > 0 \longleftrightarrow \widehat{\alpha}^K > \widehat{\alpha}^P \longleftrightarrow \frac{\alpha^K}{\alpha^P} > \frac{1-\tau^K}{1-\tau^P}$

▸ Symmetric mobility

Analytical Example: Pareto Income and HSV Taxes

$$y^G \sim \text{Pareto}(\alpha^G) \quad \longrightarrow \quad \text{Gini Coefficient: } \mathcal{G}(Y^G) = \frac{1}{2\alpha^G - 1}$$

- Taxes are progressive so that disposable income is log-linear in market income

(Feldstein 1969; Benabou 2002; Heathcote, Storesletten, Violante 2014)

$$\widehat{y}_i^G = \lambda^G (y_i^G)^{1-\tau^G} \quad \longrightarrow \quad \widehat{\alpha}^G = \frac{\alpha^G}{1-\tau^G}$$

Net mobility: $\widehat{\widehat{m}} > 0 \quad \longleftrightarrow \quad \widehat{\alpha}^K > \widehat{\alpha}^P \quad \longleftrightarrow \quad \frac{\alpha^K}{\alpha^P} > \frac{1-\tau^K}{1-\tau^P}$

▸ Symmetric mobility

- Children's progressivity (τ^K) raises mobility; Parents' progressivity (τ^P) lowers it.
- Common progressivity ($\tau^K = \tau^P$) preserves mobility sign and attenuates magnitude

Bounding Mobility

Maximal Average Mobility

Objective: Express realized mobility as a fraction of *attainable* mobility.

- Makes different mobility measures comparable

Maximal net mobility

$$\overline{m} = \frac{1}{2}\left[\mathcal{G}(Y^P) - \mathcal{G}(Y^K)\right] \longrightarrow \sup \overline{m} = \frac{1}{2}$$

- Maximal attained going from full concentration of income to full equality

- Given the parental distribution: $\overline{m}^s \leq \mathcal{G}(Y^P)/2$

- Given the child distribution: $\overline{m}^s \leq [1 - \mathcal{G}(Y^K)]/2$

Maximal Average Mobility

Objective: Express realized mobility as a fraction of *attainable* mobility.

- Makes different mobility measures comparable

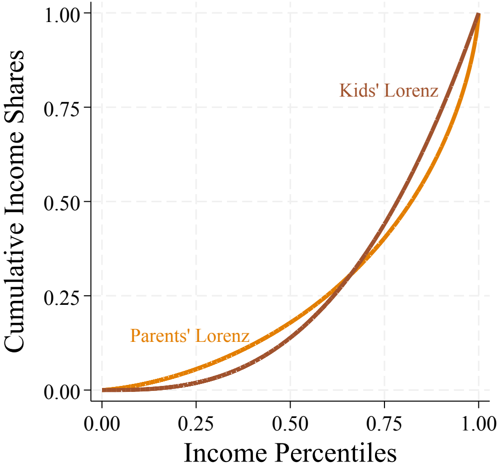
Maximal net mobility

$$\overline{m} = \frac{1}{2}\left[\mathcal{G}(Y^P) - \mathcal{G}(Y^K)\right] \longrightarrow \sup \overline{m} = \frac{1}{2}$$

- Maximal attained going from full concentration of income to full equality
- Given the parental distribution: $\overline{m}^s \leq \mathcal{G}(Y^P)/2$
- Given the child distribution: $\overline{m}^s \leq [1 - \mathcal{G}(Y^K)]/2$

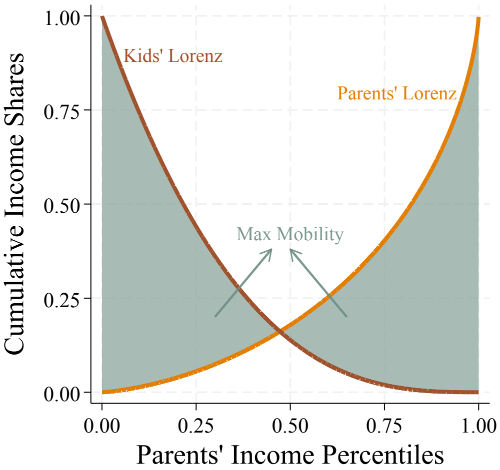
Maximal Symmetric mobility: Rank-dependence + Shape of Lorenz curves

Maximal Symmetric Mobility



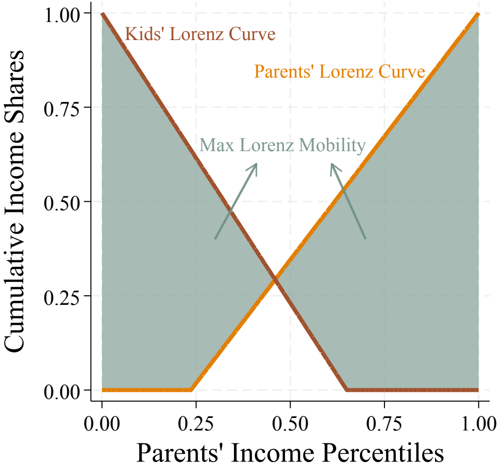
1. Vertical and horizontal mobility matter

Maximal Symmetric Mobility



1. Vertical <u>and</u> horizontal mobility matter
2. Counter-monotone ranks maximize horizontal mobility for fixed Lorenz
   - Establish M satisfies the *Monge property*

Maximal Symmetric Mobility



1. Vertical and horizontal mobility matter

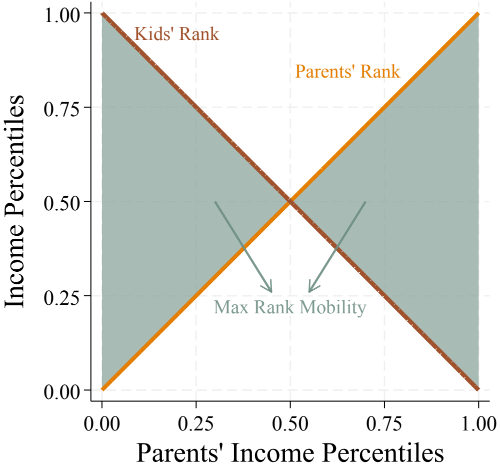
2. Counter-monotone ranks maximize horizontal mobility for fixed Lorenz

   - Establish M satisfies the *Monge property*

3. Extremal Lorenz curves maximize over distributions (Baillo, Carcamo, Mora-Corral 2022)

$$0 \leq \mathcal{M} \leq 2 - \sqrt{2} \approx 0.586$$

Maximal Rank Mobility



1. Vertical and horizontal mobility matter
2. Counter-monotone ranks maximize horizontal mobility for fixed Lorenz
   - Establish M satisfies the *Monge property*
3. Extremal Lorenz curves maximize over distributions (Baillo, Carcamo, Mora-Corral 2022)

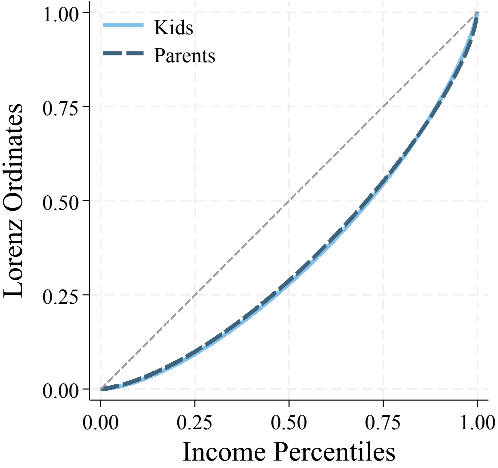
$$0 \leq \mathcal{M} \leq 2 - \sqrt{2} \approx 0.586$$

Maximal rank mobility:

$$0 \leq \frac{1}{N}\sum_{i=1}^{N}\left| r_i^K - r_i^P \right| \leq \frac{1}{2}$$

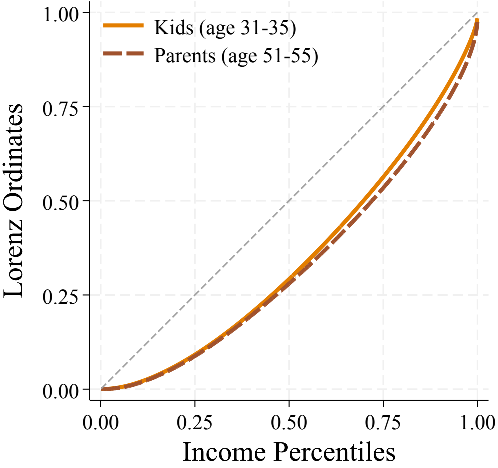
Measuring Intergenerational Mobility in the US and Norway

United States



Linked NLS66–NLSY79 parents and children
(Davis and Mazumder, 2024)

Norway



Linked parent-child tax records
Norwegian administrative data

▸ Income distributions

Signed Mobility and Inequality

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} (m_i)^\gamma \right)^{1/\gamma}$$

		Signed mobility		
		$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$
US	**Lorenz**	-0.01	-0.0	-0.05
	Rank	–	0.0	-0.05
Norway	**Lorenz**	0.02	0.0	0.09
	Rank	–	0.0	-0.04

Rank mobility averages to zero when $\gamma = 1$ by construction.

- US inequality ↑: Mobility $<$ **0**
- NOR inequality ↓: Mobility $>$ **0**

Signed Mobility and Inequality

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} (m_i)^{\gamma} \right)^{1/\gamma}$$

		Signed mobility		
		$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$
US	**Lorenz**	-0.01	-0.0	-0.05
	Rank	–	0.0	-0.05
Norway	**Lorenz**	0.02	0.0	0.09
	Rank	–	0.0	-0.04

Rank mobility averages to zero when $\gamma = 1$ by construction.

- US inequality ↑: Mobility $<$ **0**
- NOR inequality ↓: Mobility $>$ **0**
- Mobility is concentrated: $\gamma = 1/2$ gives values near zero

Signed Mobility and Inequality

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} (m_i)^\gamma \right)^{1/\gamma}$$

		Signed mobility		
		$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$
US	**Lorenz**	-0.01	-0.0	-0.05
	Rank	–	0.0	-0.05
Norway	**Lorenz**	0.02	0.0	0.09
	Rank	–	0.0	-0.04

Rank mobility averages to zero when $\gamma = 1$ by construction.

- US inequality ↑: Mobility $< \mathbf{0}$
- NOR inequality ↓: Mobility $> \mathbf{0}$
- Mobility is concentrated: $\gamma = 1/2$ gives values near zero
- Large ΔLorenz in NOR are ↑ even when large Δrank are ↓

Signed Mobility and Inequality

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} (m_i)^\gamma \right)^{1/\gamma}$$

		Signed mobility		
		$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$
US	**Lorenz**	-0.01	-0.0	-0.05
	Rank	–	0.0	-0.05
Norway	**Lorenz**	0.02	0.0	0.09
	Rank	–	0.0	-0.04

Rank mobility averages to zero when $\gamma = 1$ by construction.

- US inequality ↑: Mobility $<$ **0**
- NOR inequality ↓: Mobility $>$ **0**
- Mobility is concentrated: $\gamma = 1/2$ gives values near zero
- Large ΔLorenz in NOR are ↑ even when large Δrank are ↓
- In US, large movements are ↓ in ranks and Lorenz ordinates

Signed Mobility and Inequality

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} (m_i)^{\gamma} \right)^{1/\gamma}$$

		Signed mobility		
		$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$
US	**Lorenz**	-0.01	-0.0	-0.05
	Rank	–	0.0	-0.05
Norway	**Lorenz**	0.02	0.0	0.09
	Rank	–	0.0	-0.04

Rank mobility averages to zero when $\gamma = 1$ by construction.

- US inequality ↑: Mobility $< \mathbf{0}$
- NOR inequality ↓: Mobility $> \mathbf{0}$
- Mobility is concentrated: $\gamma = 1/2$ gives values near zero
- Large ΔLorenz in NOR are ↑ even when large Δrank are ↓
- In US, large movements are ↓ in ranks and Lorenz ordinates
- Taxes increase mobility in NOR

Symmetric Mobility: Lorenz mobility is lower than rank mobility

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} (m_i)^{\gamma} \right)^{1/\gamma}$$

		Symmetric mobility		
		$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$
US	**Lorenz**	0.26	0.21	0.34
	Rank	0.28	0.23	0.35
Norway	**Lorenz**	0.28	0.23	0.35
	Rank	0.30	0.25	0.38

- Lorenz mobility is lower for every γ in both countries

▸ Intergenerational Persistence

- Relative to attainable mobility:
  - US: Lorenz 44%, ranks 56%
  - Norway: Lorenz 47%, ranks 60%

  Lorenz $\approx$ 10pp lower than rank
- Consistent with rank changes in flatter portion of Lorenz curves

Conclusions

Lorenz mobility combines

1. Ordinal information of ranks
2. Cardinal information of income

Provides a measure of mobility that responds to changes in income and inequality

- Inequality determines how positional movements translate into mobility
- Net Lorenz mobility is the change in the Gini coefficient and is panel independent
- Distinguishes positional reshuffling and captures changes in inequality and taxation

Appendix

Lorenz Ordinates as Aspirational Status

Let status depend on own income and the income distribution: $s(y, Y)$.

- **Monotonicity:** status rises with own income
- **Growth independence:** common rescaling does not change status
- **Aspirational status:** status rises as a dynasty approaches those above and moves away from those below
- **Anonymity:** only cumulative income above and below matters, not identities

Lorenz Ordinates as Aspirational Status

Let status depend on own income and the income distribution: $s(y, Y)$.

- **Monotonicity:** status rises with own income
- **Growth independence:** common rescaling does not change status
- **Aspirational status:** status rises as a dynasty approaches those above and moves away from those below
- **Anonymity:** only cumulative income above and below matters, not identities

$$s(y, Y) = \psi\big(r(y, Y), L(y, Y)\big),$$

where ψ is increasing in rank and strictly increasing in the Lorenz ordinate.

Back

Status Units

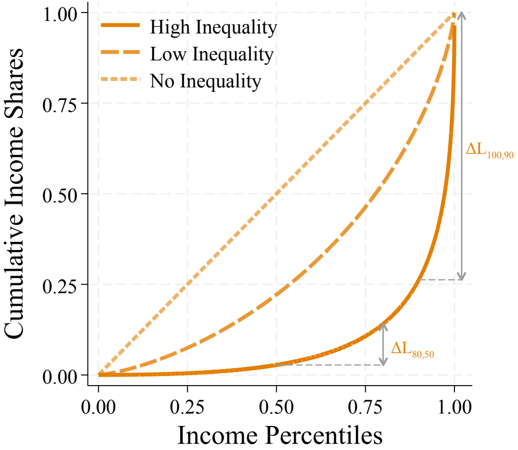
Requiring status to move linearly along rays connecting co-monotone distributions gives

$$s(y_i, Y) = \Omega(r_i)\left[\lambda L_i + (1-\lambda)\Xi(r_i)\right], \qquad \lambda \in (0,1].$$

- Lorenz ordinates provide the cardinal content of status
- Increasing rank weights $\Omega(r)$ allow status to depend on position
- $\lambda = 1$ and $\Omega(r) = 1$ give Lorenz ordinates
- Ranks arise as the limiting case $\lambda \longrightarrow 0$, $\Omega(r) = 1$, and $\Xi(r) = r$

Back

Redistribution: Proportional Transfers



$$Y^{\alpha} = \alpha Y + (1-\alpha)\mu\mathbb{1}, \qquad \alpha \in [0,1].$$

Ranks are unchanged, but

$$L_i^{\alpha} = \alpha L_i + (1-\alpha) r_i \geq L_i.$$

- Every dynasty's Lorenz ordinate weakly rises
- Above-average dynasties can lose income while moving upward in Lorenz status
- Redistribution moves the entire curve toward ranks

◂ Back

Signed Mobility Aggregation

Back

$$\mathcal{M}_N = \text{sign}\left(\frac{1}{N}\sum_{i=1}^{N}\text{sign}(m_i)|m_i|^\gamma\right)\left|\frac{1}{N}\sum_{i=1}^{N}\text{sign}(m_i)|m_i|^\gamma\right|^{1/\gamma}.$$

Upward and downward mobility

$$\mathcal{U} \equiv \frac{1}{N}\sum_{i=1}^{N}|m_i|^+, \qquad \mathcal{D} \equiv \frac{1}{N}\sum_{i=1}^{N}|m_i|^-, \qquad \overline{m} = \mathcal{U} - \mathcal{D}.$$

Decomposing aggregate mobility: Net mobility and mobility concentration

$$\mathcal{M}_N = \underbrace{\overline{m}}_{\text{Net Mobility}} \times \underbrace{\frac{\text{sign}(\mathcal{S}_\gamma)}{\text{sign}(\overline{m})}}_{\gamma\text{-Sign Correction}} \times \underbrace{\overline{\mathcal{C}}(M)}_{\text{Net Mobility Concentration}}$$

$$\mathcal{S}_\gamma(M) \equiv \sum_{i=1}^{N}\text{sign}(m_i)|m_i|^\gamma \qquad \overline{\mathcal{C}}(M) \equiv \left|\left(\frac{\mathcal{U}}{|\overline{m}|}\mathcal{C}_\gamma^+(M)\right)^\gamma - \left(\frac{\mathcal{D}}{|\overline{m}|}\mathcal{C}_\gamma^-(M)\right)^\gamma\right|^{1/\gamma}.$$

Mobility Concentration

Back

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} m_i^\gamma \right)^{1/\gamma}$$

Curvature γ controls role of concentration

$$\mathcal{C}_\gamma(M) = \exp\left(\frac{\gamma - 1}{\gamma} D_\gamma(M) \right).$$

$D_\gamma(M)$ is the Rényi entropy measure (dispersion relative to uniform distribution)

$$D_\gamma(M) = \log N - H_\gamma(r) \qquad \text{where } H_\gamma(r) = \tfrac{1}{1-\gamma} \log \left(\sum_{i=1}^{N} \left(\left| \frac{m_i}{N\overline{m}} \right| \right)^\gamma \right)$$

$\gamma < 1$ **:** penalizes concentrated mobility $\gamma > 1$ **:** favors large, concentrated movements

Mobility Concentration

Back

$$\mathcal{M}_N = \left(\frac{1}{N} \sum_{i=1}^{N} m_i^\gamma \right)^{1/\gamma}$$

Curvature γ controls role of concentration

$$\mathcal{C}_\gamma(M) = \exp\left(\frac{\gamma - 1}{\gamma} D_\gamma(M) \right).$$

$D_\gamma(M)$ is the Rényi entropy measure (dispersion relative to uniform distribution)

$$D_\gamma(M) = \log N - H_\gamma(r) \qquad \text{where } H_\gamma(r) = \tfrac{1}{1-\gamma} \log \left(\sum_{i=1}^{N} \left(\left| \frac{m_i}{N\overline{m}} \right| \right)^\gamma \right)$$

$\gamma < 1$: penalizes concentrated mobility $\quad$ $\gamma > 1$: favors large, concentrated movements

Mobility Concentration

◂ Back

$$\mathcal{M}_N = \left(\frac{1}{N}\sum_{i=1}^{N} m_i^{\gamma}\right)^{1/\gamma} = \underbrace{\overline{m}}_{\text{Average Mobility}} \times \overbrace{\mathcal{C}_{\gamma}(M)}^{\text{Mobility Concentration}}$$

Curvature γ controls role of concentration

$$\mathcal{C}_{\gamma}(M) = \exp\left(\frac{\gamma - 1}{\gamma} D_{\gamma}(M)\right).$$

$D_{\gamma}(M)$ is the Rényi entropy measure (dispersion relative to uniform distribution)

$$D_{\gamma}(M) = \log N - H_{\gamma}(r) \qquad \text{where } H_{\gamma}(r) = \tfrac{1}{1-\gamma} \log\left(\sum_{i=1}^{N}\left(\left|\frac{m_i}{N\overline{m}}\right|\right)^{\gamma}\right)$$

$\gamma < 1$: penalizes concentrated mobility $\quad$ $\gamma > 1$: favors large, concentrated movements

Average Symmetric Mobility

Define average upward and downward signed mobility by $\mathcal{U}^s$ and $\mathcal{D}^s$.

$$\overline{m}^a = \frac{1}{2}\left|\mathcal{G}(Y^P) - \mathcal{G}(Y^K)\right| + 2\left[\mathcal{D}^s \mathbb{1}\{\mathcal{G}(Y^P) \geq \mathcal{G}(Y^K)\} + \mathcal{U}^s \mathbb{1}\{\mathcal{G}(Y^P) < \mathcal{G}(Y^K)\}\right]$$

- First term: absolute net vertical mobility
- Remaining term: positional reshuffling not summarized by the Gini
- If inequality falls, total upward mobility equals the net gain plus the downward movements that it offsets

Back

Progressive Taxation and Rank Dependence

Under Pareto incomes and an FGM copula for parental and child ranks,

$$C_\theta(u, v) \;=\; uv\,[1 + \theta(1-u)(1-v)]\,, \qquad \theta \in [-1, 1].$$

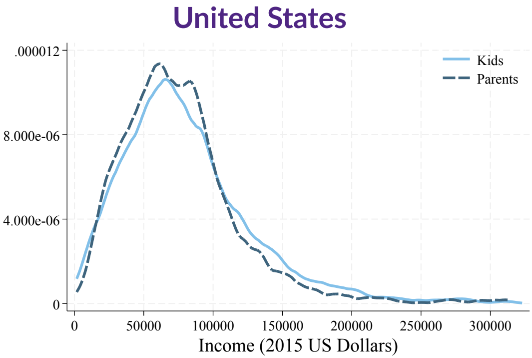
Let $p_G = \alpha_G/(\alpha_G - 1 + \tau_G)$ denote after-tax Lorenz curvature. Then

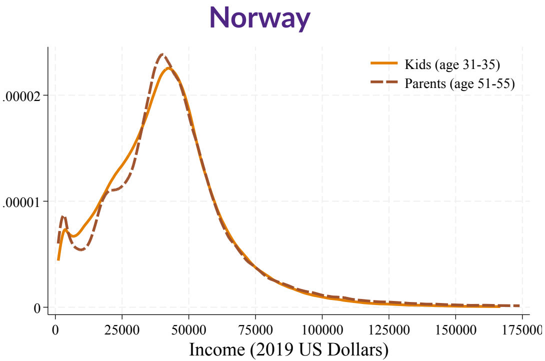
$$\mathcal{M}_N \;\propto\; \frac{p_K}{p_K + 1} + \frac{p_P}{p_P + 1} - 2\int_0^1 C_\theta\left(1 - t^{p_K}, 1 - t^{p_P}\right) dt.$$

- The effect of common progressivity is affine in rank dependence θ
- Progressivity raises symmetric mobility only below a positive dependence threshold
- With stronger dependence, horizontal mobility fades and inequality compression dominates

Back

Income Distributions Across Generations





- US children have 7.8% higher average income than their parents
- Norwegian children have 7.6% lower average income than their parents
- Lorenz mobility removes these changes in aggregate income scale

Back

Mobility in Disposable Income: Norway

	Signed mobility			Symmetric mobility		
	$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$	$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$
Lorenz	0.031	0.002	0.129	0.277	0.232	0.350
Rank	–	0.000	−0.036	0.298	0.250	0.374

- Taxes and transfers increase signed Lorenz mobility relative to market income
- Symmetric Lorenz and rank mobility are largely unchanged

Back

Intergenerational Persistence: Similar Results

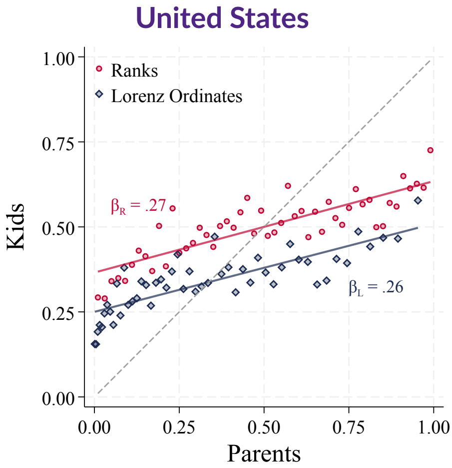
Galtonian Regression: $x_i^K = \alpha_x + \boldsymbol{\beta_x} x_i^P + u_i$ Mobility: $1 - \widehat{\boldsymbol{\beta}}_{\boldsymbol{x}}$

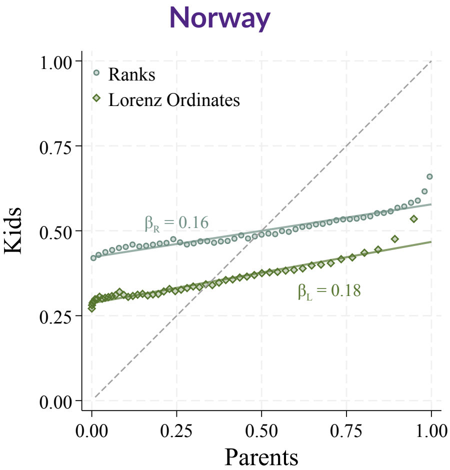
	Lorenz	Rank	Log-Lorenz	Log-income
US	0.26	0.23	0.27	0.30
Norway	0.18	0.16	0.10	0.11

- Greater persistence (lower mobility) in the US under every measure
- Rank implies somewhat more mobility than Lorenz measures (income somewhat less)
- Ultimately estimates are quantitatively close (McGee 2026)

Binned scatter plots Back

Intergenerational Regressions: Levels



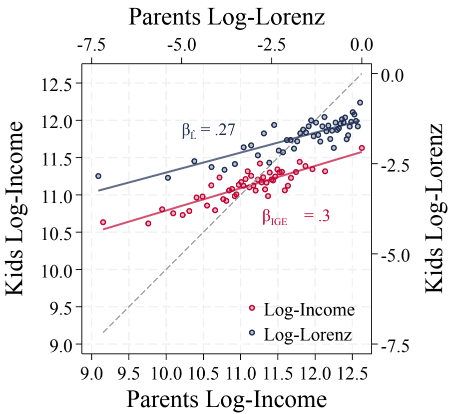


Binned means and linear fits for ranks and Lorenz ordinates across generations.

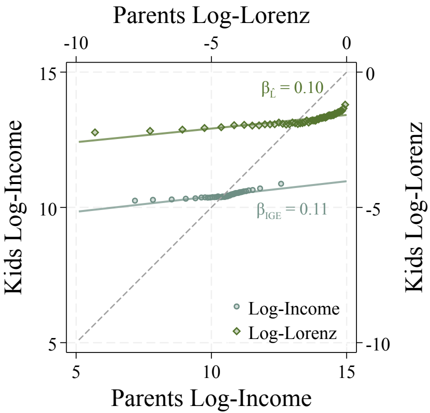
Back

Intergenerational Regressions: Logs

United States



Norway



Log-Lorenz and log-income regressions use separate axes

Back