Macroeconomics, Problem Set 2

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The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia or matlab script that produces the results.

1. Do exercise 10.4 (a), (c), (e), (f) of SLP [Optional: Continue with Exercise 13.4 of SLP]

2. Consider the stochastic version of the neoclassical growth model at the beginning of Section 8 of the lecture notes.

   (a) Write down an algorithm for how to solve the RCE using the planner's problem.

   This is just the pseudo code. Similar to the algorithms in the notes. The objective is to know the precise steps you would take to find the RCE.

   (b) Write down an algorithm for how to solve the planner's problem. Assume that the planner faces a discrete choice over capital, so that capital can only take values on a grid $\vec{k} = [k_1, \ldots, k_{n_k}]$.

   This is just the pseudo code. Similar to the algorithms in the notes. The objective is to know the precise steps you would take to find the RCE.

   (c) Assume that $\log z_{t+1} = \rho \log z_t + \epsilon_{t+1}$ with $\epsilon_t \overset{iid}{\sim} N\left(0, \sigma^2\right)$.

      i. Construct a discrete Markov process for productivity with $n_z = 5$ states using either Tauchen's or Rowenhurst's methods. Set values of $\rho = 0.9$ and $\sigma = 0.50$. [Hint: Use the notes and code from my comptuational economics class, lecture 5].

      ii. Find the stationary distribution of z using the transition matrix.

      iii. Then simulate a Markov chain with 10000 periods and drop the first 2000. Use the remaining periods to construct a histogram for z. Compare the histogram with the stationary distribution in a graph.

(d) Implement your algorithm for solving the planner's problem.

   i. Set $n_k = 150$ with $k_1 = 0.5 \times k_{ss}$ and $k_{n_k} = 1.5 \times k_{ss}$, where k_{ss} is the steady state level of capital if $z = 1$ (you can find it from the Euler equation, evaluating when $z = 1$ and there are no shocks, so no expected value). You might have to adjust the bounds of the capital grid depending on the solution.

   ii. Report the resulting value function, policy functions, and Euler residuals.

   Hint: Use the slides and code from Lecture 2 of the computational macro course.

(e) Construct a Markov process for the state of the economy and find its stationary distribution using the transition matrix. [Hint: You can either use the eigenvalues or iterate on some initial distribution].

(f) Then simulate a Markov chain with 10000 periods and drop the first 2000. Use the remaining periods to construct a histogram for (k, z). Compare the histogram with the stationary distribution of k in a graph.

(g) Compare the unconditional distribution and for the conditional distribution of k given that $z = z_1$, $z = z_3$, and $z = z_5$.

3. Recursive Competitive Equilibrium

   There is an economy with many identical agents with preferences given by

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \alpha \left(1 - n_t\right)^{\frac{1}{2}} + \gamma P_t^{\frac{1}{2}} \right]$$

   where c_t is their own consumption at time t, n_t is the fraction of their own time worked at time t,and P_t are public parks. Their initial wealth is A . The technology to produce output uses capital (that depreciates at rate δ) and labor: $Y_t = F\left(K_t, N_t\right)$.

   (a) What conditions would be satisfied in a Pareto Optimum in steady state? [Hint: Use the planner's problem to obtain these conditions.]

   Imagine now that the government levies income taxes and issues debt to pay for the parks. Its initial debt is B.

(b) Define an RCE for this economy. That is, define (recursively) the set of private and government policies that constitute an equilibrium together with all the necessary elements (like prices).

Imagine now that this is a small open economy and borrowing and lending can occur a and sell at the international rate $\bar{r}$.

(c) Define Recursive competitive equilibrium for this case and for the appropriate policies.

(d) Give an expression for the wage, and for the stock of capital in this equilibrium.

4. **[Optional]** Read Arellano (2008) before solving this problem. Consider a default model under of a small open economy. Every period there is a possible state of the world $s_t \in S$. Denote $s^t = (s_1, \ldots, s_t) \in S^t$ the history of states up and including date t. Denote the country's endowment $y(s)$. The preference of this small open economy is represented by

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

International financial markets are imperfect. First, the small open economy can only borrow or lend state-uncontingent bond $b(s)$. Second, the country can choose to default, with $d(s) = 1$ denoting default, and $d(s) = 0$ denoting non-default. After default, its debt is written off, but the country enters financial autarky. **Under financial autarky, the country cannot borrow from international market but it can save secretly with world interest rate R.** In addition, its endowment becomes $h(y)$ when the country stays at financial autarky. With probability λ, the country regains the access to international financial markets.

Given the country's option to default, international lenders incorporates the country's default risk and charge a country specific bond price.

(a) Define a recursive equilibrium for this problem.

(b) Prove that default decision is non-increasing in current bond holding.

(c) Prove that country will not choose to default if it holds positive assets ($b > 0$).

(d) [Super Optional] Solve the recursive equilibrium under the parameter values in Arellano (2008). You can make up any parameter values you do not find.

(e) [Super Optional] Plot default area in a graph with endowment at x-axis and bond at y-axis. Plot the bond price schedule for the smallest endowment and for the largest endowment as a function of current loan demand.

(f) [Super Optional] Simulate the model 10000 times, and report the corresponding statistics as in Table 4 of Arellano (2008). In addition, report the default probability, maximum and minimum of the interest rate spread, average current account-over-GDP ratio $|CA|/y$.