Macroeconomics, Problem Set 3

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The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia or matlab script that produces the results.

1 Income fluctuations and the cost of inflation

This question is based on İmrohoroğlu (1992) at the Journal of Economics Dynamics and Control. You should read the paper before tackling the questions.

Consider an economy with a continuum of infinitely-lived agents who seek to maximize the expected discounted value of their utility,

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \right].$$

These agents have (common) preferences for consumption and supply labor inelastically. Their income in consumption units, ϵ, fluctuates stochastically following a Markov process with transition function Q. These are idiosyncratic fluctuations independent across agents.

The consumption good cannot be stored. But the agents have access to paper money, printed by the government. Money cannot be consumed, but it can be stored. People in this economy are extremely distrustful of one another, and so they cannot sign contracts with one another (financial or otherwise). They are worried that if they sign a contract that stipulates future payments their counterpart will default on them. The outcome of this is that there are no financial markets in this economy.

New money is printed by the government at a rate g, so that $M_t = (1+g)\, M_{t-1}$ is the amount of money in period t. The government transfers all the new money to the agents lump-sum.

The price of consumption goods in terms of money units is p_t. Let $1+\pi_t = {}^{p_t}/_{p_{t-1}}$ denote the inflation rate (the growth rate of the price).

1. What is the state vector of an agent?

2. What is the nominal budget constraint of an agent? (nominal refers to the constraint being written in terms of money units). What is the real budget constraint? (real refers to the constraint being written in terms of consumption units, this requires you to introduce notation for "real money balances")

3. Write down the dynamic programming problem of an agent. Use the real budget constraint for this.

4. Write down the agents' Euler equation. Interpret in terms of the saving motives of the agents. What do they depend on? In particular, how are they affected by inflation?

5. Define the law of motion of the distribution of agents using the policy functions from the agents' dynamic problem and explain how to obtain the stationary distribution of agents.

6. How many markets are there in this economy? Write market clearing conditions for them. Doing this requires introducing notation for the distribution of agents over the states defined in part (1)

7. Define a stationary recursive competitive equilibrium for this economy. Use the real budget constraint for this.

8. Why did we have to use the real budget constraint to define the S-RCE? Explain

9. In the S-RCE the real money balances are constant because there is no aggregate risk. Explain. Then, use this fact (together with aggregation and market clearing) to show that the equilibrium inflation rate constant and is equal to the growth rate of money, $\pi_t = g$.

10. Write down an algorithm to solve the S-RCE in the computer if the problem is discretized (so that all choices are discrete and the Markov processes are too).

2 Capital-income risk

This question is based on Angeletos (2007) at the Review of Economic Dynamics.

Consider an economy with a continuum of infinitely-lived entrepreneurs. Each entrepreneur supplies (inelastically) one unit of labor in the market and also has a firm. These entrepreneurial firms produce using a common constant-returns-to-scale technology that employs capital and labor. Entrepreneurs are subject to idiosyncratic productivity shocks, z_i, that affect their production. The technology of production for firm i is $y_i = z_i f(k_i, \ell_i)$. The entrepreneurial productivity z_i is drawn every period from a distribution ψ (independently across entrepreneurs). The average productivity across agents is always 1 ($E[z_i] = \int z\psi(z)\,dz = 1$).

Crucially, the entrepreneurs must choose capital before they know the realized value of their productivity z_i.

Entrepreneurs can hire labor in the market at a wage rate w, but cannot rent capital, so entrepreneurs can only invest in the firm they own [the reason is that the productivity of each entrepreneur is private information, and entrepreneurs would not know what type of firm they are investing in if they were to lend capital to another firm]. There is, however a non-state-contingent bond that the entrepreneurs can use to save and borrow, call it b_i. This is a pure financial asset in zero net supply in the economy. The (gross) rate of return on the bond is R.

Entrepreneurs seek to maximize the expected discounted value of their utility. However, they have recursive Epstein-Zin preferences, instead of the usual time-additive preferences with CRRA utility. The recursive preferences they have exhibit constant elasticity of inter-temporal substitution (CEIS) and constant relative risk aversion (CRRA):

$$u_t = U(c_t) + \beta U\left(\mathrm{CE}\left[U^{-1}(u_{t+1})\right]\right),$$

where the certainty equivalent of the future utility u_{t+1} is $\mathrm{CE}[x] = \Upsilon^{-1}(E[\Upsilon(x)])$. The utility functions U and Υ aggregate consumption across dates and states, respectively; they are given by

$$U(c) = \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} \qquad \Upsilon(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

where θ is the elasticity of inter-temporal substitution and γ is the coefficient of relative risk aversion.

The recursive problem of an entrepreneur is

$$V_t(s) = \max\ U(c) + \beta U\left(\Upsilon^{-1}\left(\int \Upsilon\left(U^{-1}\left(V_{t+1}\left(s^{'}\right)\right)\right)\psi(z)\,dz^{'}\right)\right)$$

where s is the state vector. Note that the value function depends on time because the aggregates of the economy can change (deterministically) over time. Choosing the state is as always crucial.

1. Define a recursive competitive equilibrium for this economy. Assume in your definition that prices evolve over deterministically over time.

2. Pose the profit maximization problem of the entrepreneurial firm. Recall that this problem takes as given the level of capital. Solve the problem to obtain the optimal labor demand and the optimal profit level of an entrepreneur. This is Lemma 1 of the paper. You can solve for things generally, or you can assume that $f(k,\ell) = k^{\alpha}\ell^{1-\alpha}$ and solve explicitly.

3. Write down the budget constraint of the entrepreneurs in terms of its portfolio choice over capital and bonds.

4. Define the financial wealth of the household (equation 7 in the paper) and write the entrepreneurs' dynamic problem in terms of it.

5. Solve the portfolio allocation problem of the entrepreneurs using guess-and-verify. This is Lemma 2 of the paper. You can solve for things generally or assume that $\theta = 1/\gamma$ to simplify the recursive problem of the entrepreneurs.

   (a) Interpret the variables ζ, ϕ, and ρ that characterize the solution.

   (b) Provide simplified expressions for the case in which $\theta = 1/\gamma$. Interpret the optimal choice of the entrepreneur.

6. Show that the economy admits exact aggregation. That is, find expressions for the aggregate variables and prices in terms of the solution to the entrepreneurs' problem. This is

proposition 1 of the paper. You can simplify results if you assume that $\theta = 1$.

7. Focus now on the steady state of the model, when aggregate variables and prices do not change over time. Prove Propostion 2.

8. Explain equations (22) and (23) in your own words.

9. Explain why it must be that $R\beta < 1$ and $f_K(K,1) \geq R$ in steady state .

10. Explain the result in Proposition 3 and the intuition behind it.