Macroeconomics, Problem Set 4

Sergio Ocampo Díaz

The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia or matlab script that produces the results.

1 A closed-economy with span of control

Consider a static (one period) economy with infinitely many individuals. Individuals differ in their entrepreneurial (or managerial) talent denoted by z_i. Assume that talent is distributed Pareto with minimum value of $\underline{z} = 1$ and a Pareto coefficient of ξ, so that the CDF of the distribution is $\Pr(\tilde{z} \leq z) = 1 - z^{-\xi}$.

Individuals also have an endowment of assets that differs across agents and is distributed independently of their entrepreneurial talent. Assume that the aggregate assets are $\overline{K} = E[a_i] > 0$. There is a market for capital where individuals can lend their assets to firms at a market rate r.

Individuals face an occupational choice. They can use their entrepreneurial talent to start a firm, doing so has a fixed cost $\psi > 0$. If they start a firm they produce final goods with a technology $z_i g(F(k_i, n_i))$, where $g(x) = x^{\alpha}$ $\alpha \in (0,1)$ and $F(k,n) = k^{\gamma} n^{1-\gamma}$. If they do not start a firm they can work. Each individual has one unit of time that they offer inelastically. The market wage is w.

Individuals value consumption. The capital and labor markets are perfectly competitive.

1. Pose the profit maximization of an entrepreneur with talent z. Solve the problem. That is, find functions $\pi^{\star}(z)$, $n^{\star}(z)$, $k^{\star}(z)$ that characterize the optimal profits, and labor and capital demand of a entrepreneur. Take prices r and w as given.

2. Pose the occupational choice problem of individuals.

3. Define an equilibrium for this economy.

4. Solve the equilibrium in this economy. In particular, solve for the prices that clear the markets. You can make any assumptions you deem useful for this (for instance you can do away with capital or the fixed set up costs).

5. How does the distribution of firm productivity (talent) differ from the distribution of talent in the population?

6. How does talent translate into differences in income? For this compare the distribution of profits and the distribution of talent among entrepreneurs. In particular, if an entrepreneur has 10 times more talent than another, how much more income (profits) do they have?

7. How does the distribution of firm size by employment differ from the distribution by capital. Interpret the difference.

8. Analyze the solution. What do the comparative statics of the equilibrium prices and quantities tell you about the economy.

2 Equilibrium in an economy with differentiated goods

Consider a static (one period) closed economy where firms produce differentiated products as in Dixit & Stiglitz. Consumers have preferences for a bundle of goods according to

$$U = \left(\int y_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

1. Let $R = PY$ be the total spending (or total revenue), P be an idea price index and Y the total "aggregate quantity" in the market. P and Y satisfy $PY = \int p_i y_i di = \int r_i di$. Derive the optimal demand curve for y_i in terms of p_i, Y, and P and the revenue (or spending) in good i r_i in terms of total revenue R, p_i, P. Derive an expression for the ideal price index P as a function of the prices of individual varieties.

   The production of goods uses only labor. There are L units of labor in the economy. We use labor as the numeraire so that $w = 1$ and all prices are expressed in terms of units of labor. (so a price p means that someones would have to work p units of labor to pay for the good).

   Firms differ in their productivity z that determines their marginal costs, $1/z$. They all have to pay a a fixed cost φ to setup production for the local market.

2. Pose the profit maximization problem of a firm with productivity z_i that is choosing the price of their good subject to the demand curve they face. The firm takes as given aggregates. Find expressions for the firm's revenue $r(z)$ and the firms' profits $\pi(z)$ (including fixed costs).

3. Some times we cannot observe prices and quantities separately in the data. We instead observe revenue and cost-of-goods-sold. This gives rise to a measure of productivity given by the ratio of revenue to costs.

   (a) Calculate this measure and argue whether or not it is a good proxy for the actual productivity of the firm.

Similarly, if we only observe revenue we do not know if firms with higher revenue produce more or less (in quantities).

(a) Compare the ratio of revenues and the ratio of quantities produced of two firms with $z_i > z_j$. What can you conclude about quantities and productivities if you see that a firm has higher revenue than another firm?

In equilibrium (to be characterized later) there is a distribution of firm productivity $z \in (0, \infty)$ with pdf $\mu(z)$ and a mass of $M > 0$.

4. Express the price aggregator in terms of an integral with respect to productivity (z) and not to firm indices. This is possible because your previous results show that all firm with the same productivity charge the same price. The density of firms with productivity z is $M\mu(z)$. Then, use the optimal pricing of firms to express the aggregate price P as a function of M and the average productivity, $\bar{z}$. Show that the relevant measure of the average is a curved-weighted harmonic mean $\bar{x} = \left(\int x^{\varepsilon-1}\mu(x)\,dx\right)^{\frac{1}{\varepsilon-1}}$.

   (a) Show you can do the same for revenue, R, total profits, Π, and total quantity, Y.

We now want to figure out what determines μ and M. This comes from the entry and exit decisions of firms. The modeling of this follows Hopenhayn (1992). There is a large pool of identical potential entrants deciding whether to become active or not. Firms deciding to become active pay a fixed cost of entry $\varphi_e > 0$ (this is different from the operating cost φ above) and then get a productivity draw z from a CDF Γ. After observing their productivity draws, firms decide whether to remain active or not.

Assume that Γ is Pareto, so that $\Gamma(z) = 1 - \left(\frac{z}{\underline{z}}\right)^{-\xi}$ for some $\underline{z} > 0$, and a pdf $\gamma(z) = \xi\underline{z}^{\xi}z^{-(\xi+1)}$

5. Characterize the operating decision of firms in terms of cutoff for their productivity, $z^\star$. What can you say about it?

Having the cutoff for operations gives us the distribution of operating firms

$$\mu(z) = \begin{cases} \frac{\gamma(z)}{1-\Gamma(z^\star)} & \text{if } z \geq z^\star \\ 0 & \text{otw} \end{cases}$$

6. Use this information to solve for the average productivity of operating firms $\overline{z}$.

   There is free entry. This means that, in equilibrium, firms will enter as long as their expected profits cover the fixed entry cost φ_e. Recall that firms do not their productivity before they enter. If they draw a productivity below $z^\star$ they get no profits, if they draw a productivity above $z^\star$ (with probability $1 - \Gamma(z^\star)$) they get the average profits of surviving firms: $\overline{\pi} = \frac{\Pi}{M}$. You solved for Π above.

7. State the free entry condition of the firms and explain what it means for the equilibrium level of average profits if you increase the cost of entry.

8. The solution to the model is the pair of values $z^\star$ and $\overline{\pi}$ that simultaneously satisfy the free entry condition and the zero-profit condition for the marginal operating firm. Show that the profits of the marginally operating firm are increasing in productivity and that the average profits are constant with respect to the productivity of the marginal operating firm. Draw a diagram with the free entry and the zero-profit curves as functions of productivity to characterize the solution.

   The last step to close the model is to clear the goods and labor market. The free entry condition implies that aggregate profits are zero after paying for entry costs. So, to clear the goods market it must be that $R = L$, that is, total revenue (obtained by firms) must be equal to the total income of consumers (recall that the wage is normalized to 1).

9. Use the market clearing condition to obtain an expression for the mass of firms M in terms of size of the economy L, the elasticity ε and the profits of firms. Relate the solution to the taste for variety.

3 International trade: The Melitz model

Consider the same model as in the previous question. We are now going to open the economy. A firm has to pay a fixed cost φ_x in order to export. Moreover, exporting goods is more expensive than selling goods in the local market. This is reflected in a higher marginal cost of τ/z per unit of good exported, $\tau > 1$. These are called iceberg costs in the literature because they are equivalent to having to send τ units of goods for every unit of successful exports (so that $\tau - 1$ goods do not reach the destination, they sink while at sea!). People in other countries have the same preferences for goods as in the local country and have firms with the same technology. The solution across countries is symmetric and so if a firm in the "domestic" economy exports, there is an equivalent firm in the foreign economy that is also exporting.

1. If a firm exports they can charge a different price in the local and the foreign market. What is the optimal price for exported goods?

2. Firms export if doing so increases their profits. Characterize a cutoff for productivity $z_x^\star$ above which firms export. Assume that $\tau^{\varepsilon-1}\varphi_x > \varphi$. Show that this implies that $z_x^\star > z^\star$ so that only some of the firms who operate export. You can show this graphically with a diagram of profits ($\pi_x(z)$ and $\pi_d(z)$). You can show in the diagram the three regions for productivity (firms do not operate, they operate but don't export, they export). The total profits of firms are $\pi = \pi_d + \pi_x$.

3. Show that if $\tau^{\varepsilon-1}\varphi_x > \varphi$ then exporting firms have higher measured productivity (revenue over costs) than domestic firms.

4. Compute the aggregates in the economy (P, R, Π, Y) as functions of the average productivity among all firms $\bar{z} = \left(\frac{1}{M}\left(M_d\bar{z}_d^{\varepsilon-1} + M_x\bar{z}_x^{\varepsilon-1}\right)\right)^{\frac{1}{\varepsilon-1}}$, where $M = M_d + M_x$ is the total number of varieties (domestic+foreign). $\bar{z}_d = \left(\frac{1}{1-\Gamma(z^\star)}\int_{z^\star}^{\infty} z^{\varepsilon-1}\gamma(z)\,dz\right)^{\frac{1}{\varepsilon-1}}$ is the average productivity among domestic producers and $\bar{z}_x = \left(\frac{1}{1-\Gamma(z_x^\star)}\int_{z_x^\star}^{\infty} z^{\varepsilon-1}\gamma(z)\,dz\right)^{\frac{1}{\varepsilon-1}}$ is the average productivity among exporters.

   The free entry condition works the same as before, but now the average profits include the likelihood of having a high enough draw to become an exporter. $\overline{\pi} = \overline{\pi}_d + \frac{1-\Gamma(z_x^\star)}{1-\Gamma(z^\star)}\overline{\pi}_x$, where $\frac{1-\Gamma(z_x^\star)}{1-\Gamma(z^\star)}$ is the conditional probability of being an exporter if the firm operates. The free entry condition is $0 \times \Gamma(z^\star) + (1 - \Gamma(z^\star))\,\overline{\pi} = \varphi_e$.

5. Use the definition of the cutoffs $(\pi_d(z^\star) = \varphi; \pi_x(z_x^\star) = \varphi_x)$ to express $z_x^\star$ as a function of $z^\star$ and the fixed costs.

6. Use these results to show that the zero profit curve with trade lies above the zero profit curve in autarky (the closed economy above). Interpret what that implies for the value of the cutoff productivity $z^\star$ and the consequences of opening to trade.