

Lorenz Mobility^{*}

Rory M^cGee Sergio Ocampo

September 2025

Abstract

An individual's Lorenz ordinate—their position in the Lorenz curve—can be interpreted as their *aspirational* economic status. We formalize this interpretation by providing an axiomatic derivation of economic status and its resulting mobility that combines the ordinal content of ranks with the cardinal content of differences in income. The key feature captured by Lorenz ordinates is that status is *upward looking* and increases when an individual's income is closer to those richer than them, regardless of the income of those poorer. The units of economic status and mobility are readily interpretable as income shares comparable across time. In this way, horizontal (or positional) mobility is only meaningful if there are material differences between incomes across the distribution, directly tying inequality to mobility. We show how the resulting mobility relates to other measures of mobility and to standard concepts of income inequality, and provide an application to intergenerational status mobility by sex using Norwegian administrative data.

^{*}M^cGee: UWO & IFS; rmcgee4@uwo.ca; Web: <https://sites.google.com/view/rorymcgee>.

Ocampo: UWO; socampod@uwo.ca; Web: <https://sites.google.com/site/sergiocampod>.

Ocampo acknowledges financial support from the Research Council of Norway through the project TaxFair, Number 315765. M^cGee and Ocampo acknowledge financial support from the Social Science and Humanities Research Council of Canada through the Insight Development Grant Number 430-2022-00394.

1. Introduction

The studies of mobility and inequality are intertwined, with many measures of economic mobility focusing on the *relative* position of individuals in the economy. These include intergenerational changes in income ranks (Bartholomew 1973 and Shorrocks 1978b), and rank correlations (Schiller 1977, Chetty et al. 2014 or Fagereng, Mogstad, and Rønning 2021).¹ However, purely ordinal measures obscure the connection between mobility and inequality by discarding cardinal information in income. Without cardinality, relative mobility measures struggle to capture the material differences between individuals.

The interaction between ordinal inequality measures and cardinal income differences is illustrated by how ranks relate to income concentration along the Lorenz curve. Consider Figure 1A, which shows the Lorenz curve of the log-Normal distribution with increasing inequality.² In more unequal economies, income distributions compress at the bottom, where there is little material difference between the incomes of the lowest-ranked individuals, and spread at the top, where differences are large. As inequality decreases, income differences between ranks shrink until ranks become equidistant in income. An individual's Lorenz ordinate—their position in the Lorenz curve—captures this relationship: differences in Lorenz ordinates shrink when income differences between ranks are small and spread as these differences increase.

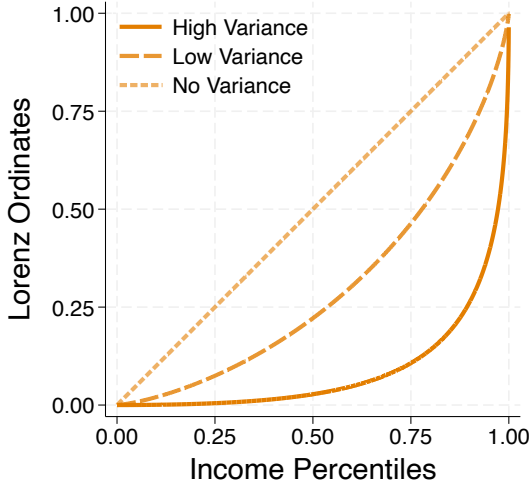
We show that Lorenz ordinates provide a natural measure of *aspirational* economic status, which explicitly connects changes in mobility and inequality. This characterization arises under two key conditions. First, status is *aspirational* in the sense that it only depends on the income of richer individuals: individuals are

¹While most of the discussion in this paper applies to different economic outcomes, such as wealth, we focus on income throughout for comparability with the literature and ease of exposition.

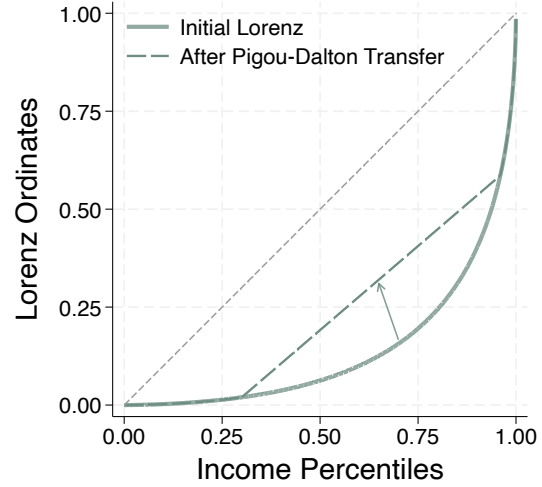
²The same results apply to other distributions, like the Pareto distribution with tail parameter α , whose Lorenz curve is $L(r) = 1 - (1 - r)^{\frac{\alpha-1}{\alpha}}$.

FIGURE 1. Lorenz Mobility

(A) Income Ranks and Income Differences



(B) Pigou-Dalton Transfers



Notes: Panel A shows the Lorenz curves—cumulative income shares at given percentiles—for three log-normal distributions with different degrees of dispersion or, equivalently, after successive proportional Pigou-Dalton transfers. The Lorenz curve of a log-normal distribution is $L(r) = \Phi(\Phi^{-1}(r) - \sigma)$, where Φ is the standard normal cumulative density and σ^2 is the variance. The no-variance line corresponds to the limit as $\sigma \rightarrow 0$. Panel B shows a Lorenz curve before and after Pigou-Dalton transfers.

upward-looking, reflecting the objective of catching up to those above. Second, status satisfies an *anonymity* property with respect to others: it depends on how rich they are, not who they are or how income is distributed among them. The result is that an individual's status is determined by the share of income held by those richer than them, increasing when there is less (income) distance to the top. Then, we axiomatize individual mobility, taking as given a measure of economic status, and economy-wide mobility, following the standard for quasi-arithmetic aggregators.³

Our approach explicitly clarifies the implications of inequality for mobility. Changes in status decompose into *horizontal mobility*, capturing changes in the position of individuals holding inequality constant (hence moving along the Lorenz curve), and *vertical mobility*, capturing changes in inequality that shift the Lorenz curve. Changes in

³Most aggregate mobility measures correspond to quasi-arithmetic aggregators, see, for instance, Bartholomew (1973); Fields and Ok (1996, 1999); Cowell and Flachaire (2018); Ray and Genicot (2023).

position are meaningful in as much as inequality makes their Lorenz ordinates different (Figure 1A). Shifts in the Lorenz curve that reduce inequality, such as those coming from Pigou-Dalton transfers, increase status (Figure 1B). The link between inequality and mobility is complete when aggregating signed changes in status. Then, mobility is fully characterized by changes in the Gini coefficient.⁴ Our approach also disentangles overall economic growth from status and mobility whose units are expressed in income shares and are comparable across economies.

We connect our Lorenz mobility measure to existing approaches by separating the axiomatization of aggregation, mobility, and status. Our measure of status can be used to compute aggregate absolute mobility (Fields and Ok 1996, 1999), signed mobility (Bhattacharya and Mazumder 2011; Cowell and Flachaire 2018; Ray and Genicot 2023), and intergenerational correlations (Hart 1983).⁵ The justification for these measures, such as the axiomatic derivation in Shorrocks (1993) for Hart’s mobility measure, applies equally to mobility in status.⁶

While not driven by equity concerns, our results share properties with theories of inequality aversion (Robson 1992) and fairness (Fehr and Schmidt 1999), where the position of an individual in the economy is relevant (see also Lazear and Rosen 1981). However, position is not sufficient as an increase in rank does not affect status without an increase in income, underscoring the role of income for the consumption capabilities of individuals (Sen 1973). Our characterization of individual status via Lorenz ordinates is also linked to indexes of aggregate *satisfaction* in the context of relative deprivation (e.g., Runciman 1966; Yitzhaki 1979; Kakwani 1984). These indexes aggregate income

⁴These results highlight that status and mobility are not separable from the distribution of income, linking to extensive work interpreting mobility as social welfare (Atkinson and Bourguignon 1982).

⁵These are commonly implemented by computing the intergenerational elasticity of income (e.g., Solon 1992; Zimmerman 1992; Dearden, Machin, and Reed 1997; Mazumder 2005; Chetty et al. 2014; or Bolt, French, Hentall-MacCuish, and O’Dea 2024) or the Spearman correlation of income ranks (e.g., Schiller 1977; Dahl and DeLeire 2008; Chetty et al. 2014; Olivetti and Paserman 2015; Fagereng, Mogstad, and Rønning 2021; Ward 2023; or Jácóme, Kuziemko, and Naidu 2025).

⁶In a separate context, Audoly et al. (2025) use Lorenz ordinates to study wealth mobility as an alternative to ranks and log-wealth that emphasizes top wealth holders.

comparisons between individuals, as the axiomatization in [Ebert and Moyes \(2000\)](#) makes clear, while status is conceptually different, capturing an individual's capabilities and position in society.

Finally, we apply this framework to intergenerational status mobility by sex using administrative tax data covering the entire Norwegian population 1993–2017 (see [Blundell, Graber, and Mogstad 2015](#) for details on the Norwegian income tax records). Focusing on Norwegians born between 1961 and 1970, we find status mobility between mothers and daughters exceeds mobility between fathers and sons, driven entirely by higher horizontal mobility among women. Despite women having nearly identical Lorenz curves across generations (hence having no vertical mobility), changes in position generate status mobility equivalent to 29% of aggregate income, 3 percentage points higher than for men.

2. Lorenz Mobility

Consider an economy populated by a finite set of dynasties indexed $i = 1, \dots, N$. There are two observations of income for each dynasty that we call y^P and y^K , these might be incomes of parents and their children or incomes of the same individual at different points in time. We denote by Y^P and Y^K the $(N \times 1)$ vectors of income.

2.1. Status

We are interested in mobility with respect to the *status* of individuals. We see status as capturing the consumption capabilities of individuals (e.g., [Sen 1973](#); [Yitzhaki 1979](#)) as well as their position in the economy. Thus, we assume status is a function of an individual's income and the income distribution in the economy, $s : \mathbb{R}_+ \times \mathbb{R}_+^N \longrightarrow \mathbb{R}_+$.

We constrain how status depends on income by imposing 4 conditions expressed in Axioms 1–4 below. We start by normalizing the scale of status, forcing it to be bounded.

AXIOM 1 (**Status Normalization**). *Status satisfies $s(y, Y) \in [0, 1]$.*

We restrict attention to status functions that are strictly monotone in the dynasty's income and right-continuous. Monotonicity is key to distinguish purely ordinal representations of status, like income ranks, from cardinal representations that reflect income levels. Specifically, status increases with the dynasty's income, holding the rest of the income distribution constant. Right-continuity accounts for ties in the finite economy leading to jumps in status when income changes so as to break those ties.

AXIOM 2 (**Continuity and Monotonicity**). *Status $s(y_i, Y)$ is right-continuous and strictly increasing on y_i given $\{y_j\}$ for $j \neq i$.*

In order to disentangle economic growth from economic mobility we focus on status functions that capture *relative* as opposed to *absolute* mobility. Specifically, we preclude changes in status from changes in the scale of income. Therefore, status depends only on the shape of Y and the relative position of y in Y , but not on their level. Moreover, as in [Shorrocks \(1993\)](#), this imposes *scale invariance* within and between generations, making redundant the units in which income is measured.

AXIOM 3 (**Growth Independence**). *Status satisfies $s(y, Y) = s(\kappa y, \kappa Y)$ for any $\kappa > 0$.*

Finally, we pin down how status responds to the shape of the income distribution. As stated above, our objective is that status combines cardinal information reflecting differences in income levels with ordinal information reflecting relative positions in the distribution. Specifically, we impose that the status of a dynasty must be invariant to pure redistribution above or below them. Clearly this redistribution leaves the dynasty's rank unaltered highlighting the positional content of status. This makes status “*anonymous*,” a given dynasty does not care about the identities of other dynasties (whose positions can be reshuffled by redistribution) while still caring about their own relative position. This property makes the cumulative distribution of income the relevant statistic for determining status.

We strengthen this notion by making status *aspirational*, with dynasties focusing on those who are richer than them. We see this as reflecting the objective of a dynasty to catch up with (or take from) those above them. Status increases when the dynasty gets closer (in income) to those above, not by looking at those below. So, only redistribution from those who are richer increase status.⁷

AXIOM 4 (Aspirational Status). Order dynasties in Y such that $y_i \leq y_{i+1}$ for all $i = 1, \dots, N$. Let $1 < h < j \leq N$ be two dynasties and make any transfer δ between them such that

$$y_{h-1} < y_h + \delta < y_{j+1} \quad \text{and} \quad y_{h-1} < y_j - \delta < y_{j+1} \quad (1)$$

letting $y_{j+1} = y_j$ if $j = N$. The transfer δ can be positive or negative. Then

- (a) The status of dynasties $i = 1, \dots, h-1, j+1, \dots, N$ remains unchanged: $s(y_i, Y) = s(y_i, Y^\delta)$ where Y^δ is the same as Y except for the incomes of the h and j entries; and
- (b) The status of dynasty j is unchanged if their rank is unchanged: if $y_{j-1} < y_j - \delta < y_{j+1}$, then $s(y_j, Y) = s(y_j - \delta, Y^\delta)$.

Together, these axioms tie status, and hence mobility, to the Lorenz ordinates of dynasties in the income distribution. Lorenz ordinates retain the ordinal and cardinal information in income and its distribution. Status reflects position first, with cardinality provided only if income moves to bring a dynasty closer to those they aspire to reach.

PROPOSITION 1 (Lorenz Status). A status function $s : \mathbb{R}_+ \times \mathbb{R}_+^N \rightarrow \mathbb{R}_+$ satisfies Axioms 1–4 if and only if $s(y, Y) = \psi(L(y, Y))$, for $\psi : [0, 1] \rightarrow [0, 1]$ an increasing and continuous function and L the Lorenz curve of Y ,

$$L(y, Y) \equiv \sum_{\{\tilde{y} \in Y \mid \tilde{y} \leq y\}} \frac{\tilde{y}}{N\mu}; \quad \text{where} \quad \mu \equiv \sum_{i=1}^N \frac{y_i}{N}. \quad (2)$$

PROOF. Let Y be such that $y_i \leq y_{i+1}$ for all $i = 1, \dots, N$ and fix a dynasty i . Consider an alternative $N \times 1$ ordered income vector Y' with the same average income, $\mu' = \mu$, and

⁷As in the TV series *Mad Men*, copywriters look up to Don Draper, but Draper *does not think about them at all*.

cumulative income among those richer than i , $\sum_{\{\tilde{y}' > y'_i\}} \tilde{y}' = \sum_{\{\tilde{y} > y_i\}} \tilde{y}$. Assume the vectors coincide in their i^{th} entry, $y'_i = y_i$, so the sum of income below i also coincides.

It is possible to move from Y to Y' by means of transfers among dynasties $1, \dots, i-1$ and dynasties $i+1, \dots, N$. Axiom 4 demands $s(y_i, Y) = s(y_i, Y')$. Therefore, status cannot depend on the whole distribution of income but only on y_i 's position or rank, μ , and the sum of income up to y_i (or equivalently the sum of income above).

Now consider a transfer that takes from i giving to $h < i$, without changing i 's rank. By Axiom 4, this does not change status. So, status cannot depend on y_i directly except for its effect on the cumulative sum of income up to it, $s(y, Y) = \psi\left(r(y, Y), \sum_{\{\tilde{y} > y\}} \tilde{y}, \mu\right)$.

Axiom 3 implies mean income is superfluous. Any re-scaling of income must result in the same status and so we can write $s(y, Y) = \psi\left(r(y, Y), \sum_{\{\tilde{y} > y\}} \frac{\tilde{y}}{N\mu}\right)$. The second argument is the Lorenz ordinate, which like the rank, is increasing in income. Hence, ψ must be increasing in both arguments by Axiom 2. Ranks are only weakly increasing in income, as not all changes in income change relative positions. Thus, ψ cannot depend directly on them to preserve strict monotonicity, leaving only the Lorenz ordinate as an argument. Finally, ψ must satisfy Axiom 1. \square

We further isolate how status depends on the shape of the income distribution by restricting how it moves across co-monotone distributions—where dynasties share the same ranks. Specifically, we require status moves linearly along the ray connecting any two such distributions. This condition settles how status units deal with the cardinality of income changes, as the position of each dynasty is fixed along these movements, making the dependence of status on Lorenz ordinates complete.

AXIOM 5 (Co-Monotone Rays). *Order dynasties in Y such that $y_i \leq y_{i+1}$ for all $i = 1, \dots, N$. Consider an alternative income vector Y' with the same strict order for all dynasties. An economy along the ray connecting these two economies is an income vector $Y^\alpha = \alpha Y + (1 - \alpha) Y'$, for $\alpha \in [0, 1]$. Status satisfies $s(y_i^\alpha, Y^\alpha) = \alpha s(y_i, Y) + (1 - \alpha) s(y'_i, Y')$.*

Axiom 5 only covers co-monotone mixing between income distributions. For instance, the outcome of any transfer that changes the rank of a dynasty is not covered, neither are distributions with rank reversals between dynasties. Among co-monotone distributions, the axiom imposes no restrictions on their shape. In particular, their

corresponding Lorenz curves can cross, imposing no conditions on the change in the level of status across generations (we allow $s(y_i, Y) > s(y'_i, Y')$ for some i and $s(y_j, Y) < s(y'_j, Y')$ for some $j \neq i$). Nevertheless, this condition is enough to obtain linearity of the ψ function.

LEMMA 1 (Status Units). *Let s be a status function satisfying Axioms 1–4. Status satisfies Axiom 5 if and only if $s(y, Y) = L(y, Y)$.*

PROOF. Let Y such that $y_i \leq y_{i+1}$ with strict inequality for at least one i and Y' an income vector with the same strict order. Without loss, we have $\mu = \mu'$, as status and the Lorenz curve are scale invariant (Axiom 3). Under Proposition 1, Axiom 5 requires

$$\psi(L(y_i^\alpha, Y^\alpha)) = \alpha\psi(L(y_i, Y)) + (1 - \alpha)\psi(L(y'_i, Y')). \quad (3)$$

From Aaberge (2001), the Lorenz is linear under co-monotone mixtures,

$$L(y_i^\alpha, Y^\alpha) = \alpha L(y_i, Y) + (1 - \alpha)L(y'_i, Y'). \quad (4)$$

Hence, ψ satisfies (3) if and only if it is affine. If Y and Y' are such that $y_i = y_j$ and $y'_i = y'_j$ for all i, j , Axiom 3 implies $s(y, Y) = s(y', Y') = s(y^\alpha, Y^\alpha)$, which satisfies Axiom 5 immediately. Axiom 1 then forces ψ to be the identity to keep $s \in [0, 1]$. \square

Lemma 1 pins down the mapping from income distributions to individual status, providing clear and interpretable units for our status measure (and later for mobility) in terms of *income shares*. Crucially, Lorenz ordinates (and hence status) are the same under incomes $\{Y^P, Y^K\}$ as in an economy where average income is constant across generations, $\{\mu^K/\mu^P Y^P, Y^K\}$. This separates overall income growth from mobility in economic status. Consequently, an increase in status by 0.1 for dynasty i corresponds to the dynasty increasing their own income by 10% of aggregate income in the economy (fixed at the level of income of either generation).

Inequality and status. The conditions we have imposed imply that status depends on the *Lorenz ordinates* of dynasties. This clarifies the implications of inequality for mobility:

movements towards equality increase status. This is more easily seen by exploring the effects of Pigou-Dalton transfers, which modify the shape of the income distribution locally by making it less unequal.⁸ These transfers towards equality necessarily increase status in the economy in the sense of the usual stochastic order, as they lead to a Lorenz curve that is pointwise above the original as in Figure 1B, producing an order over inequality and status (see, Atkinson 1970). Moreover, when Pigou-Dalton transfers preserve ranks, the status of each dynasty increases (strictly for at least one).

For instance, consider proportional Pigou-Dalton transfers which smoothly reduce inequality across the distribution, delivering a new vector $Y^\alpha = \alpha Y + (1 - \alpha) \mu$ for some $\alpha \in [0, 1]$.⁹ These transfers towards equality preserve ranks and thus increase the status of all dynasties (as seen in Figure 1A). The new status satisfies

$$s(y_i^\alpha, Y^\alpha) = \sum_{\{\tilde{y} \in Y \mid \tilde{y} \leq y_i\}} \frac{\alpha \tilde{y} + (1 - \alpha) \mu}{N\mu} = \alpha s(y_i, Y) + (1 - \alpha) r(y_i, Y), \quad (5)$$

where $r(y, Y) \geq L(y, Y)$ is the rank of y on Y . Thus, status is maximized under equality with $s(\mu, [\mu]_{i=1}^N) = 1$.

2.2. Status mobility

We now turn to characterizing how to measure status mobility taking the measurement of economic status as given. We impose mobility is symmetric, thus it measures the absolute change in status, similar to the measure in Fields and Ok (1996) for income levels. We discuss signed mobility in Section 3.

PROPOSITION 2 (Absolute Status Mobility). *A mobility function $m : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is*
(a) *Continuous in all arguments;*

⁸Consider two dynasties $h, j \in \{1, \dots, N\}$ with $y_h < y_j$. A transfer $\delta > 0$ from j to h is a Pigou-Dalton transfer if j is not poorer than h after the transfer, $y_h \leq y_j - \delta$.

⁹These transfers are not covered by Axiom 5 because they mix the original income distribution with an un-ranked income vector $Y' = [y'_i = \mu]$.

- (b) *Symmetric in that $m(s, s') = m(s', s)$;*
(c) *Step-additive in that $m(s, s'') = m(s, s') + m(s', s'')$ for all $s \leq s' \leq s''$; and*
(d) *Translation-invariant in that $m(s + \zeta, s' + \zeta) = m(s, s')$ for all s, s' and $\zeta > 0$;*
if and only

$$m(s^P, s^K) \propto |s^K - s^P|. \quad (6)$$

PROOF. Consider status s . By translation invariance of mobility $m(s, s + \zeta) = m(0, \zeta)$ for $\zeta \geq 0$. So, mobility does not depend on s , only on the gap ζ . Define mobility by a gap ζ as $g(\zeta) \equiv m(0, \zeta)$. g is immediately continuous because m is continuous. Moreover, mobility is additive on the gap, that is, $g(\zeta + \zeta') = g(\zeta) + g(\zeta')$. To see this let $\zeta \geq \zeta' \geq 0$, step-additivity of m gives

$$g(\zeta + \zeta') = m(0, \zeta + \zeta') = m(0, \zeta) + m(\zeta, \zeta + \zeta') = g(\zeta) + g(\zeta'). \quad (7)$$

Therefore, $g(\zeta) = \kappa \zeta$ for some $\kappa \geq 0$, because continuous additive functions are linear.

Now, consider status pair s and s' with $s \leq s'$. We have $m(s, s') = g(s' - s) = \kappa(s' - s)$. Finally, symmetry of mobility extends the characterization to all status pairs s and s' with $m(s, s') = \kappa |s' - s|$. □

Horizontal and vertical mobility. We further distinguish between changes in status coming from changes in rank or position under the same income distribution, *horizontal mobility*, and changes in status coming from changes in the distribution while holding rank constant, *vertical mobility*.

Formally, consider an economy with income vectors $\{Y^P, Y^K\}$, and write mobility as

$$m(s_i^P, s_i^K) = |s_i^K - s_i^P| = \overbrace{|s_i^K - s_i^{KP}|}^{\text{Vertical Mobility}} + \underbrace{|s_i^{KP} - s_i^P|}_{\text{Horizontal Mobility}}, \quad (8)$$

where $s_i^{KP} \equiv \tilde{L}^P(r_i^K)$ is the status of generation K under P 's income distribution while

retaining the rank they have under K 's distribution, and $\tilde{L}^G(r)$ is the Lorenz curve of generation $G \in \{P, K\}$ defined over ranks—the income share of individuals in G with rank lower than or equal to r . *Horizontal mobility*, the difference between s_i^{KP} and s_i^P , captures status changes coming from rank changes across generations, holding the income distribution constant. This corresponds to a (horizontal) move along generation P 's Lorenz curve. *Vertical mobility*, the difference between s_i^K and s_i^{KP} , captures changes in the distribution of status across generations, holding the dynasty's rank constant. This corresponds to a (vertical) move from generation P 's Lorenz curve to K 's curve.

Mobility between co-monotone generations comes only from vertical mobility and reflects changes in inequality. A decrease in inequality between generations implies positive vertical mobility: status is higher at each rank in the less unequal generation. Mobility between generations with the same distribution of income (as captured by the Lorenz curve) come only from horizontal mobility.

2.3. Aggregating mobility

Finally we aggregate mobility via a function $\mathcal{M} : \mathbb{R}^N \rightarrow \mathbb{R}$ from a vector M of dynastic mobilities, with a typical element m_i . We adopt the axiomatic characterization of general *quasi-arithmetic* aggregators in Kolmogorov-Nagumo-de Finetti's theorem (see [Hardy, Littlewood, and Pólya 1952](#), Thm. 215).

THEOREM 1 (Kolmogorov-Nagumo-de Finetti). *A family of aggregators $\mathcal{M}_N : \mathbb{R}^N \rightarrow \mathbb{R}$, indexed by N , is*

- (a) *Continuous and strictly increasing in all arguments;*
- (b) *Symmetric in that $\mathcal{M}_N(PM) = \mathcal{M}_N(M)$ for any permutation matrix P ;*
- (c) *Reflexive in that $\mathcal{M}_N(m\mathbb{1}_N) = m$ for any m ; and*
- (d) *Associative in that $\mathcal{M}_N(M) = \mathcal{M}_N([\bar{m}, \dots, \bar{m}, m_{K+1}, \dots, m_N])$, where $\bar{m} = \mathcal{M}_K([m_1, \dots, m_K])$;*

if and only if it has the following form

$$\mathcal{M}_N(M) = \Gamma^{-1} \left(\frac{1}{N} \sum_{i=1}^N \Gamma(m_i) \right), \quad (9)$$

for a continuous function $\Gamma : \mathbb{R} \longrightarrow \mathbb{R}$.

The conditions imposed by the Kolmogorov-Nagumo-de Finetti theorem are standard and are shared by additive aggregators common in the mobility literature (e.g., [Bartholomew 1973](#); [Fields and Ok 1996, 1999](#); [Cowell and Flachaire 2018](#); [Ray and Genicot 2023](#)). The first condition ensures higher mobility for any dynasty is reflected in higher aggregate mobility. The second condition ensures *anonymity*, so the indexes of dynasties are superfluous. The third condition ensures a form of consistency, so that if all dynasties have the same mobility ($m_i = m$) then aggregate mobility must coincide with this movement ($\mathcal{M}_N = m$). The fourth and final condition is the most restrictive, but it is necessary for the additivity of the aggregator (and the reason behind the *quasi-arithmetic* moniker). It implies common properties in the study of inequality and mobility such as replication invariance, so $\mathcal{M}_{\lambda N}(\hat{M}) = \mathcal{M}_N(M)$ for $\hat{M} = \mathbb{1}_\lambda \otimes M$ obtained by replicating the population $\lambda \in \mathbb{N}$ times.

We further pin down the aggregator by requiring \mathcal{M} be homogeneous of degree 1. So, if mobility is re-scaled, aggregate mobility is re-scaled in the same way. The result is standard and we state it without proof (see [Hardy, Littlewood, and Pólya 1952](#), Thm. 84).

LEMMA 2 (Homogeneous Aggregation). *Let $\mathcal{M}_N : \mathbb{R}^N \longrightarrow \mathbb{R}$ satisfy Theorem 1. \mathcal{M}_N is homogeneous of degree 1, so that $\mathcal{M}_N(\kappa M) = \kappa \mathcal{M}_N(M)$ for all $\kappa > 0$, if and only if $\Gamma(m) = m^\gamma$ for some $\gamma \neq 0$, with the $\gamma = 0$ case corresponding to the geometric average.*

The curvature of the aggregator, γ , determines the emphasis on broad-based versus concentrated mobility in just the same way as in measures of inequality aversion ([Atkinson 1970](#)) or choice under uncertainty (as \mathcal{M} also gives the family of certainty equivalents over $\{m_i\}$ under constant relative risk-aversion, [Akerberg, Hirano, and](#)

Shahriar 2017). When $\gamma = 1$ we recover the arithmetic mean, which neither favors nor penalizes dispersion in mobility. For positive γ , lower values favor broad-based mobility while higher values favor dispersion, effectively placing a higher weight on dynasties with the highest mobility. Negative values of γ place more weight on the dynasties with the lowest mobility. Although γ can tilt aggregate mobility towards the highest or lowest mobility, our representation of aggregate mobility precludes weighting who moves as there are no weights assigned to dynasties based on their identity or initial conditions.

3. Extensions

3.1. Signed Lorenz mobility

In Proposition 2, we required symmetry of the individual status mobilities leading to a measure of absolute status mobility. However, in some settings, society may value the *direction* of mobility across generations. The following characterization of mobility replaces symmetry in favor of *signed-symmetry*

PROPOSITION 3 (Signed Status Mobility). *A mobility function $m : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is*

- (a) *Continuous in all arguments;*
- (b) *Signed-symmetric in that $m(s, s') = -m(s', s)$ with $m(s, s') > 0$ for all $s < s'$;*
- (c) *Step-additive in that $m(s, s'') = m(s, s') + m(s', s'')$ for all $s \leq s' \leq s''$; and*
- (d) *Translation-invariant in that $m(s + \zeta, s' + \zeta) = m(s, s')$ for all s, s' and $\zeta > 0$;*
if and only if

$$\tilde{m}(s^P, s^K) \propto s^K - s^P. \quad (10)$$

PROOF. Consider status s and s' with $s < s'$. We have $m(s, s') = g(s' - s) = \kappa(s' - s) > 0$ from the proof of Proposition 2. Signed-symmetry extends the characterization to all status pairs s and s' with $m(s, s') = \kappa(s' - s)$ and $\kappa > 0$. \square

Since neither the characterization of status nor of the mobility aggregator depend on the symmetry of status mobility, our results yield a plug-in alternative using the signed mobility measure in (10). When mobility is aggregated uniformly, $\gamma = 1$, this yields a particularly intuitive closed form expression for *net* status mobility.

LEMMA 3 (Net Mobility). *Consider a mobility aggregator satisfying Theorem 1 and Lemma 2 with $\gamma = 1$, a mobility function satisfying Proposition 3, and a status function satisfying Axioms 1–5. Aggregate mobility has the unique form:*

$$\tilde{\mathcal{M}}_N(M) \propto G(Y^P) - G(Y^K), \quad (11)$$

where $G(Y) \equiv 1/2 - 2 \sum_{i=1}^N L(y_i, Y)$ is the Gini coefficient.

This result clarifies conditions under which changes in the Gini coefficient capture mobility across the population.¹⁰ When inequality decreases, so $G(Y^K) < G(Y^P)$, net mobility is necessarily positive, reflecting that a decrease in inequality increases the status of dynasties on net. For instance, implementing proportional Pigou-Dalton transfers lowers the Gini coefficient and unequivocally increases mobility as it increases the status of all dynasties (see equation 5 and Figure 1A).

However, comparisons are not straightforward in general. Even under arbitrary Pigou-Dalton transfers, dynasties are subject to both vertical and horizontal mobility as in (8) so that their status can go down even as the distribution of status in the economy improves. Nevertheless, Lemma 3 establishes net mobility corresponds to vertical mobility (with horizontal mobility being zero-sum) and is entirely captured by changes in the Gini coefficient,¹¹ making the link between mobility and inequality complete even when the Lorenz curves of Y^K and Y^P intersect. See Atkinson (1970) and Aaberge and Mogstad (2011) for alternative discussions of ranking intersecting Lorenz curves.

¹⁰This is distinct from Shorrocks (1978a) who measures mobility using the ratio of dynastic inequality to average inequality.

¹¹Reorder the sum in (11) to obtain the (vertical) difference in Lorenz ordinates by rank.

3.2. Intergenerational status correlation

An alternative approach to measure mobility estimates the correlation of individual characteristics across generations. Typically, researchers use intergenerational elasticities of income and the Spearman correlation of income ranks, often obtained from *Galtonian* regressions. The coefficient from log-income regressions can be derived axiomatically as a [Hart's 1983](#) mobility measure, as shown in [Shorrocks \(1993\)](#), or as the reduced form of [Becker and Tomes \(1979\)](#) models.

Our approach provides a justification for measuring intergenerational correlations of status, using Lorenz ordinates, instead of income or income ranks. Our Axioms 1–5 do not rely on the measure of mobility being used. This allows us to “port them” into alternative measures of mobility, such as [Shorrocks \(1993\)](#). The result is an extension of our analysis to the measurement of the intergenerational correlation or elasticity of status.¹² For example, computing $1 - \text{corr} \left(s_i^P, s_i^K \right)$ or estimating

$$\log \left(s_i^K \right) = \beta_0 + \beta_1 \log \left(s_i^P \right) + u_i, \quad (12)$$

where the mobility measure is given by $1 - \hat{\beta}_1$.

In practice these measures of intergenerational correlation produce similar evaluations of economic mobility. This is for two reasons. First, Lorenz ordinates across generations have the same concordance as ranks.¹³ Second, when inequality is low, the Lorenz curve is approximately linear which yields Lorenz ordinates proportional to rank. While this approximation is clearly poor for the extremes of the distribution, it becomes better near the median. To see this, consider log-normally

¹²In the elasticity case $\log \left(s \left(y_i, Y \right) \right)$ is well-defined as long as $Y > 0$; the same condition required by $\log \left(y_i \right)$. A regression approach can also be motivated as the best linear prediction of status.

¹³Concordance produces a common ordering over mobility measures ([M^cGee 2025](#)).

distributed incomes with variance σ^2 and Lorenz curve

$$\tilde{L}(r) = \Phi \left(\Phi^{-1}(r) - \sigma \right), \quad (13)$$

where Φ is the standard normal cumulative density function. This Lorenz curve is approximately linear for ranks around the median ($r = 1/2$) or for small values of σ .

4. Lorenz Mobility in Norway

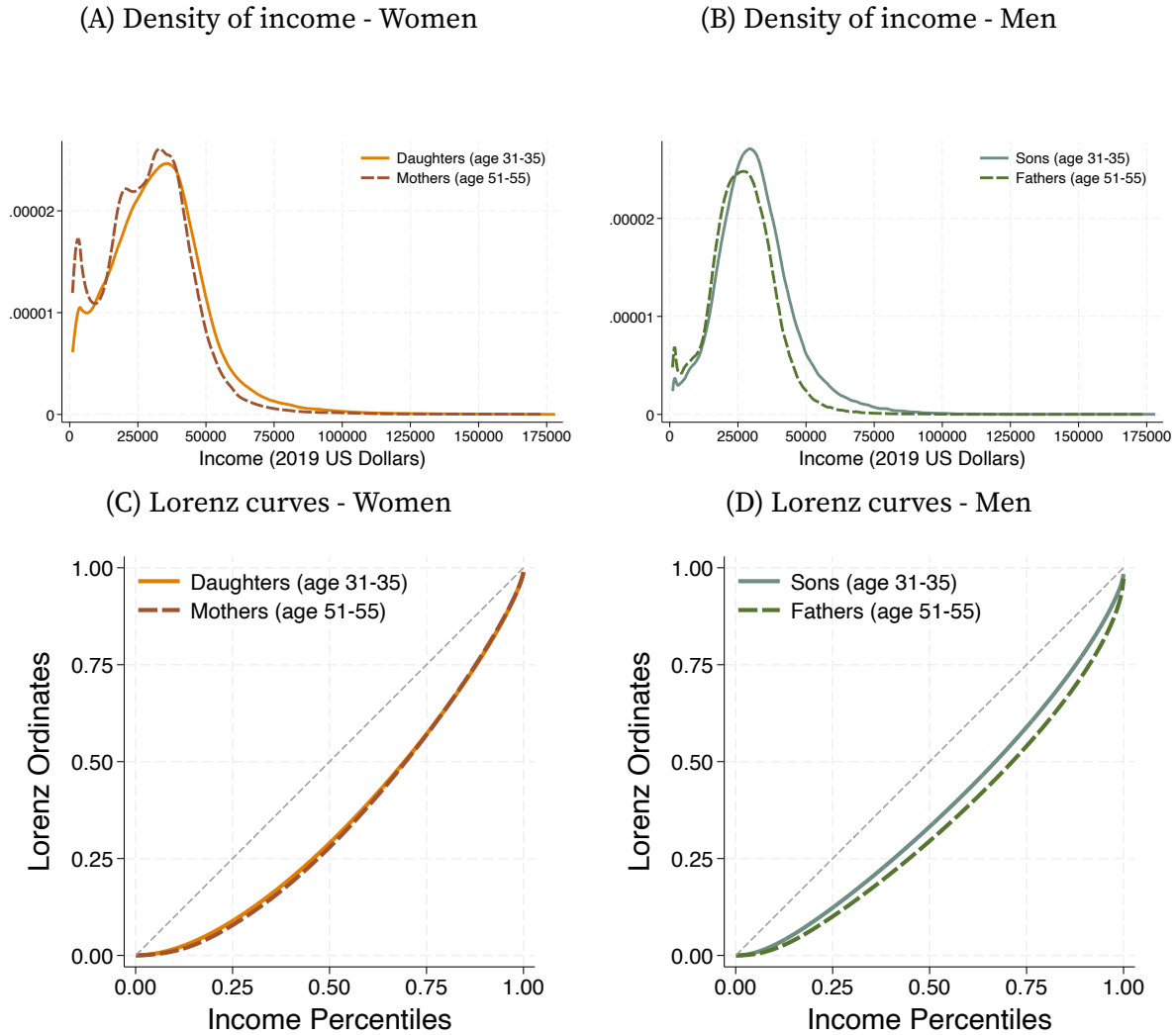
We now apply our framework to the mobility of Norwegians across generations.

Data and key variables. We use detailed longitudinal data on individual income collected by the Norwegian tax authority between 1993 and 2017 as well as demographic information available in their population files. We focus our analysis on individual pre-tax market income from wages and capital.

There are several key advantages to using the Norwegian administrative data compared with survey and administrative data available in other countries. The data covers the entire population, including the richest Norwegians, allowing us to construct accurate income ranks and Lorenz ordinates. Second, income is third-party reported, eliminating concerns about measurement error from self-reporting and censoring. Third, we link incomes across generations without relying on imputation (e.g., [Collins and Wanamaker 2022](#); [Ward 2023](#); or [Jácome, Kuziemko, and Naidu 2025](#)) or bounding exercises (e.g., [Chetty et al. 2017](#); [Berman 2022](#); or [Manduca et al. 2024](#)).

Sample selection. We study Norwegians born between 1961 and 1970 and record their average income between the ages of 31 and 35. We drop individuals who earn less than \$1,000 on average. We link these Norwegians to their fathers and mothers and record the parents' average income between the ages of 51 and 55. We construct two samples with

FIGURE 2. Income distribution across generations



Notes: Panels A and B show the kernel density of income in each generation for women and men respectively. Panels C and D show the corresponding Lorenz curves.

this data. First, a sample of mothers and daughters defined by the female Norwegians born 1961–1970 with recorded information on their mothers. This sample contains a total of 128,562 paired observations. Second, an equivalent sample for sons and their fathers with a total of 101,595 observations.

The distributions of daughters and sons move rightwards relative to their mothers and fathers, reflecting overall income growth, and have less mass concentrated at low

incomes, as seen in Figures 2A and 2B. Daughters' income is 18% higher than their mothers', with the bottom 10 percentiles of the distribution growing more than 70%. Sons' income growth is smaller, slightly below 11%, and less widespread, with the top 15 percentiles of sons actually mapping to lower income levels than their fathers'.

The changes in the shape of the income distributions across generations are reflected in the Lorenz curves. As seen in Figure 2C, there are no significant differences between the Lorenz curves of daughters and mothers, despite crossing at the top of the income distribution. In contrast, the Lorenz curve of sons shifts towards equality, Figure 2D, reflecting the higher income levels of the bottom 10 percentiles and lower income levels of the top 15.

Intergenerational status mobility. We find higher mobility in economic status for women than men across most mobility measures presented in Table 1. As we see from our main mobility measure (\mathcal{M}), women's status across generations moves on average by 29% of their aggregate income compared to men's 26%. Status mobility among women corresponds mostly to horizontal mobility, reflecting changes in the position of dynasties, as there are only small differences between the Lorenz curves of mothers and daughters (Figure 2C). The lack of vertical mobility is further evidenced by the lack of net (or Gini) mobility ($\tilde{\mathcal{M}}$).

Mobility of economic status is broad-based for women and men alike. We can see this by comparing our main measure of aggregate mobility \mathcal{M} without curvature, $\gamma = 1$, to measures with lower and higher curvature. When we set $\gamma = 1/2$, aggregate mobility favors a more even distribution of mobility, reducing \mathcal{M} if mobility is concentrated among few dynasties. However, mobility remains high, with women's status moving by 23% of total income and men's moving by 22%. Conversely, \mathcal{M} increases when setting $\gamma = 2$, as it weights more dynasties with higher mobility. These results imply that mobility is broad-based, but with some dynasties experiencing larger status changes.

TABLE 1. Intergenerational mobility measures

	Income Growth	Lorenz Mobility				Intergenerational Correlations			
		$\mathcal{M} = \left(\frac{1}{N} \sum (m_i)^\gamma \right)^{\frac{1}{\gamma}}$			$\tilde{\mathcal{M}}$	$1 - \rho \left(x_i^P, x_i^K \right)$			
		$\gamma = 1$	$\gamma = 1/2$	$\gamma = 2$		Lorenz	Log-Lorenz	Rank	Log-income
Women	0.18	0.285	0.234	0.366	0.006	0.819	0.891	0.832	0.892
Men	0.11	0.261	0.215	0.334	0.033	0.746	0.876	0.777	0.879

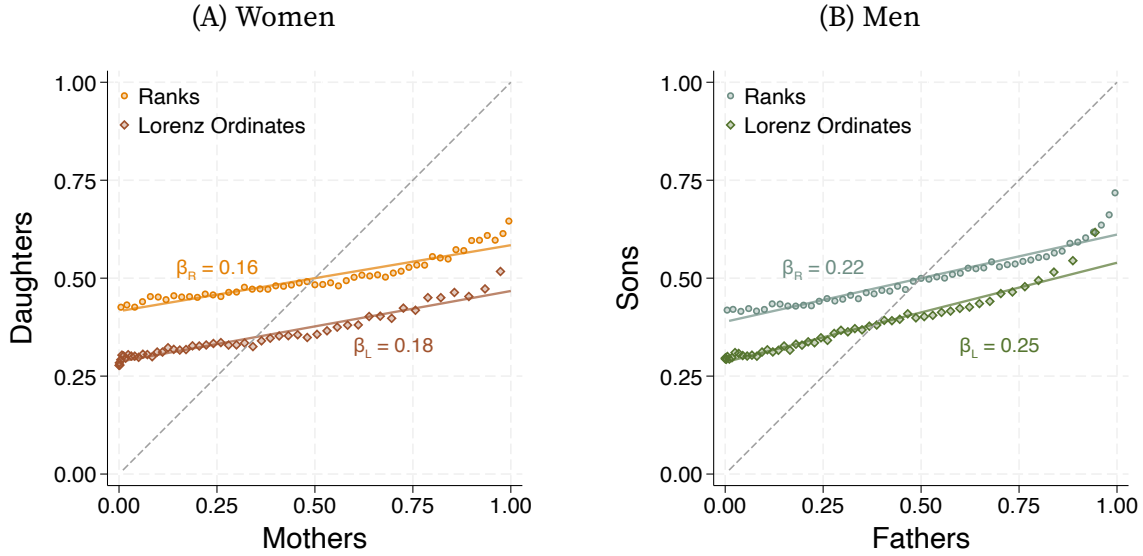
Notes: The table reports income growth along with measures of intergenerational mobility for the samples of Norwegian women (top row) and men (bottom row). Income growth is in the first column. The Lorenz mobility measures in the next 3 columns correspond to Lemma 2 and Proposition 2. The measure in the fifth column corresponds to net mobility (Lemma 3). Intergenerational correlation measures are obtained from regressions of each variable between generations.

Finally, we look at standard measures of intergenerational correlations for status, ranks, and log-income levels. We report them in the last four columns of Table 1 and the corresponding scatter plots for the cases of status and ranks in Figure 3. As expected, these different measures provide a consistent picture of intergenerational mobility (see M^cGee 2025). In particular, the correlation of economic status and income ranks are quantitatively similar. This is because the relatively low income inequality in our application results in Lorenz curves that are approximately linear between the 25th and 75th percentiles of income. The intergenerational correlation of status is, however, higher than that of ranks, implying lower mobility in status.

5. Conclusions

We show Lorenz ordinates reflect the aspirational status of individuals. We combine this with an axiomatic approach to measure the status mobility of individual dynasties and aggregate them across the population. The connection between economic status and the Lorenz curve brings standard concepts from the study of inequality to the

FIGURE 3. Intergenerational correlations: Lorenz status and ranks



Notes: The figures show binned scatters of Lorenz status in circles and income ranks in diamonds across generations along with their corresponding trend lines for women in panel A and men in panel B.

study of mobility incorporating both ordinal and cardinal information. In sufficiently equal societies, Lorenz ordinates are close to rank-based measures of status. In unequal societies, differences in Lorenz ordinates are small where the income distribution is compressed and large where income is dispersed, capturing material differences between individuals. The link between inequality and mobility is complete in a special case of net status mobility, which is equivalent to changes in the Gini coefficient.

References

- Aaberge, Rolf. 2001. "Axiomatic characterization of the Gini coefficient and Lorenz curve orderings." *Journal of Economic Theory* 101 (1):115–132. URL <https://doi.org/10.1006/jeth.2000.2749>.
- Aaberge, Rolf and Magne Mogstad. 2011. "Robust inequality comparisons." *The Journal of Economic Inequality* 9 (3):353–371. URL <https://doi.org/10.1007/s10888-010-9163-y>.
- Ackerberg, Daniel, Keisuke Hirano, and Quazi Shahriar. 2017. "Identification of time and risk preferences in buy price auctions." *Quantitative Economics* 8 (3):809–849. URL <https://doi.org/10.3982/QE469>.
- Atkinson, Anthony B. 1970. "On the measurement of inequality." *Journal of Economic Theory* 2 (3):244–263. URL [https://doi.org/10.1016/0022-0531\(70\)90039-6](https://doi.org/10.1016/0022-0531(70)90039-6).
- Atkinson, Anthony B and François Bourguignon. 1982. "The comparison of multi-dimensioned distributions of economic status." *The Review of Economic Studies* 49 (2):183–201. URL <https://doi.org/10.2307/2297269>.
- Audoly, Richard, Rory McGee, Sergio Ocampo, and Gonzalo Paz Pardo. 2025. "The life-cycle dynamics of wealth mobility." NY FED Staff Report 1097, Federal Reserve Bank of New York. URL <https://doi.org/10.59576/sr.1097>.
- Bartholomew, David J. 1973. *Stochastic models for social processes*. John Wiley & Sons.
- Becker, Gary S. and Nigel Tomes. 1979. "An equilibrium theory of the distribution of income and intergenerational mobility." *Journal of Political Economy* 87 (6):1153–1189. URL <http://www.jstor.org/stable/1833328>.
- Berman, Yonatan. 2022. "The long-run evolution of absolute intergenerational mobility." *American Economic Journal: Applied Economics* 14 (3):61–83. URL <https://doi.org/10.1257/app.20200631>.
- Bhattacharya, Debopam and Bhashkar Mazumder. 2011. "A nonparametric analysis of black-white differences in intergenerational income mobility in the United States." *Quantitative Economics* 2 (3):335–379. URL <https://doi.org/10.3982/QE69>.
- Blundell, Richard, Michael Graber, and Magne Mogstad. 2015. "Labor income dynamics and the insurance from taxes, transfers, and the family." *Journal of Public Economics* 127:58–73. URL <https://doi.org/10.1016/j.jpubeco.2014.04.011>.
- Bolt, Uta, Eric French, Jamie Hentall-MacCuish, and Cormac O'Dea. 2024. "The intergenerational elasticity of earnings: exploring the mechanisms." Tech. Rep. 25, IFS Working Papers. URL <https://ifs.org.uk/sites/default/files/2024-06/WP202425-The-intergenerational-elasticity-of-earnings-exploring-the-mechanisms.pdf>.
- Chetty, Raj, David Grusky, Maximilian Hell, Nathaniel Hendren, Robert Manduca, and Jimmy Narang. 2017. "The fading American dream: Trends in absolute income mobility since 1940." *Science* 356 (6336):398–406. URL <https://doi.org/10.1126/science.aal4617>.
- Chetty, Raj, Nathaniel Hendren, Patrick Kline, Emmanuel Saez, and Nicholas Turner. 2014. "Is the United States still a land of opportunity? Recent trends in intergenerational mobility." *American Economic Review* 104 (5):141–47. URL <https://doi.org/10.1257/aer.104.5.141>.
- Collins, William J and Marianne H Wanamaker. 2022. "African American intergenerational

- economic mobility since 1880.” *American Economic Journal: Applied Economics* 14 (3):84–117. URL <https://doi.org/10.1257/app.20170656>.
- Cowell, Frank A. and Emmanuel Flachaire. 2018. “Measuring mobility.” *Quantitative Economics* 9 (2):865–901. URL <https://doi.org/10.3982/QE512>.
- Dahl, Molly W and Thomas DeLeire. 2008. “The association between children’s earnings and fathers’ lifetime earnings: estimates using administrative data.” Tech. Rep. 1342–08, Institute for Research on Poverty, University of Wisconsin-Madison. URL <https://www.irp.wisc.edu/publications/dps/pdfs/dp134208.pdf>.
- Dearden, Lorraine, Stephen Machin, and Howard Reed. 1997. “Intergenerational mobility in Britain.” *The Economic Journal* 107 (440):47–66. URL <https://www.jstor.org/stable/2235270>.
- Ebert, Udo and Patrick Moyes. 2000. “An axiomatic characterization of Yitzhaki’s index of individual deprivation.” *Economics Letters* 68 (3):263–270. URL [https://doi.org/10.1016/S0165-1765\(00\)00248-2](https://doi.org/10.1016/S0165-1765(00)00248-2).
- Fagereng, Andreas, Magne Mogstad, and Marte Rønning. 2021. “Why do wealthy parents have wealthy children?” *Journal of Political Economy* 129 (3):703–756. URL <https://doi.org/10.1086/712446>.
- Fehr, Ernst and Klaus M. Schmidt. 1999. “A theory of fairness, competition, and cooperation.” *The Quarterly Journal of Economics* 114 (3):817–868. URL <https://doi.org/10.1162/003355399556151>.
- Fields, Gary S. and Efe A. Ok. 1996. “The meaning and measurement of income mobility.” *Journal of Economic Theory* 71 (2):349–377. URL <https://doi.org/10.1006/jeth.1996.0125>.
- . 1999. “Measuring movement of incomes.” *Economica* 66 (264):455–471. URL <https://doi.org/10.1111/1468-0335.00183>.
- Hardy, G. H., J. E. Littlewood, and G. Pólya. 1952. *Inequalities*. Cambridge: Cambridge University Press, 2nd ed.
- Hart, Peter E. 1983. “The size mobility of earnings.” *Oxford Bulletin of Economics & Statistics* 45 (2):181–193. URL <https://doi.org/10.1111/j.1468-0084.1983.mp45002003.x>.
- Jácome, Elisa, Ilyana Kuziemko, and Suresh Naidu. 2025. “Mobility for all: Representative intergenerational mobility estimates over the twentieth century.” *Journal of Political Economy* 133 (1):306–354. URL <https://doi.org/10.1086/732527>.
- Kakwani, Nanak. 1984. “The relative deprivation curve and its applications.” *Journal of Business & Economic Statistics* 2 (4):384–394. URL <https://doi.org/10.2307/1391280>.
- Lazear, Edward P. and Sherwin Rosen. 1981. “Rank-Order tournaments as optimum labor contracts.” *Journal of Political Economy* 89 (5):841–864. URL <https://doi.org/10.1086/261010>.
- Manduca, Robert, Maximilian Hell, Adrian Adermon, Jo Blanden, Espen Bratberg, Anne C. Gielen, Hans van Kippersluis, Keunbok Lee, Stephen Machin, Martin D. Munk, Martin Nybom, Yuri Ostrovsky, Sumaiya Rahman, and Outi Sirniö. 2024. “Measuring absolute income mobility: lessons from North America and Europe.” *American Economic Journal: Applied Economics* 16 (2):1–30. URL <https://doi.org/10.1257/app.20210137>.
- Mazumder, Bhashkar. 2005. “Fortunate sons: New estimates of intergenerational mobility in the United States using social security earnings data.” *Review of Economics and Statistics* 87 (2):235–255. URL <https://www.jstor.org/stable/40042900>.
- McGee, Rory. 2025. “Sufficient statistics for economic mobility.” URL <https://uctpmcg.github.io/>

- [WebsiteData/Sufficient_Statistics_for_Economic_Mobility_McGee2025.pdf](#). Working Paper.
- Olivetti, Claudia and M Daniele Paserman. 2015. "In the name of the son (and the daughter): Intergenerational mobility in the United States, 1850–1940." *American Economic Review* 105 (8):2695–2724. URL <https://doi.org/10.1257/aer.20130821>.
- Ray, Debraj and Garance Genicot. 2023. "Measuring upward mobility." *American Economic Review* 113 (11):3044–89. URL <https://doi.org/10.1257/aer.20220316>.
- Robson, Arthur J. 1992. "Status, the distribution of wealth, private and social attitudes to risk." *Econometrica* 60 (4):837–857. URL <https://doi.org/10.2307/2951568>.
- Runciman, W. G. 1966. *Relative deprivation and social justice: A study of attitudes to social inequality in twentieth-century England*. London: Routledge & K. Paul.
- Schiller, Bradley R. 1977. "Relative earnings mobility in the United States." *The American Economic Review* 67 (5):926–941. URL <https://www.jstor.org/stable/1828074>.
- Sen, Amartya. 1973. *On economic inequality*. Oxford University Press. URL <https://doi.org/10.1093/0198281935.001.0001>.
- Shorrocks, Anthony F. 1978a. "Income inequality and income mobility." *Journal of Economic Theory* 19 (2):376–393. URL [https://doi.org/10.1016/0022-0531\(78\)90101-1](https://doi.org/10.1016/0022-0531(78)90101-1).
- . 1978b. "The measurement of mobility." *Econometrica* 46 (5):1013–1024. URL <https://doi.org/10.2307/1911433>.
- . 1993. "On the Hart measure of income mobility." In *Industrial concentration and economic inequality*. Edward Elgar Publishing, 3–21.
- Solon, Gary. 1992. "Intergenerational income mobility in the United States." *The American Economic Review* 82 (3):393–408. URL <http://www.jstor.org/stable/2117312>.
- Ward, Zachary. 2023. "Intergenerational mobility in American history: Accounting for race and measurement error." *American Economic Review* 113 (12):3213–3248. URL <https://doi.org/10.1257/aer.20200292>.
- Yitzhaki, Shlomo. 1979. "Relative deprivation and the Gini coefficient." *The Quarterly Journal of Economics* 93 (2):321–324. URL <https://doi.org/10.2307/1883197>.
- Zimmerman, David J. 1992. "Regression toward mediocrity in economic stature." *The American Economic Review* 82 (3):409–429. URL <https://www.jstor.org/stable/2117313>.